Problèmes d’identification dans les graphes

Aline Parreau

Séminaire DOLPHIN

20 septembre 2012
Fire detection in a museum?

- Detector can detect fire in their room or in their neighborhood.
- Each room must contain a detector or have a detector in a neighboring room.
Fire detection in a museum?

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Modelization with a graph

- Vertices $V$: rooms
- Edges $E$: between two neighboring rooms
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- **Vertices** $V$: rooms
- **Edges** $E$: between two neighboring rooms
Modelization with a graph

- **Vertices** $V$: rooms
- **Edges** $E$: between two neighboring rooms
- **Set of detectors** = dominating set $S$:

\[ \forall u \in V, N[u] \cap S \neq \emptyset \]
Modelization with a graph

- Vertices $V$: rooms
- Edges $E$: between two neighboring rooms
- Set of detectors = dominating set $S$:

$$\forall u \in V, N[u] \cap S \neq \emptyset$$
Back to the museum
Back to the museum

Where is the fire?
Back to the museum

Where is the fire?

To locate the fire, we need more detectors.
Back to the museum

Where is the fire?
Back to the museum

Where is the fire?

To locate the fire, we need more detectors.
Identifying where is the fire
Identifying where is the fire

In each room, the set of detectors in the neighborhood is unique.
Modelization with a graph

Identifying code $C = \text{subset of vertices of a graph which is}$

- **dominating**: $\forall u \in V, N[u] \cap C \neq \emptyset$,
- **separating**: $\forall u, v \in V, N[u] \cap C \neq N[v] \cap C$.

<table>
<thead>
<tr>
<th>V \ C</th>
<th>a</th>
<th>b</th>
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<tbody>
<tr>
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</tbody>
</table>
Modelization with a graph

Identifying code \( C \) = subset of vertices of a graph which is

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\begin{center}
\begin{tabular}{c|cccc}
$V \setminus C$ & a & b & c & d \\
\hline
1 & \bullet & \bullet & - & - \\
2 & - & \bullet & - & - \\
3 & - & - & \bullet & \bullet \\
4 & - & - & - & \bullet \\
5 & \bullet & \bullet & \bullet & - \\
6 & - & \bullet & \bullet & \bullet \\
\end{tabular}
\end{center}
Modelization with a graph

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Facts about identifying codes

- Introduced in 1998 by Karpvosky, Chakrabarty and Levitin
- Motivation: fault-detection in processors networks
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- Main question:

  Given a graph $G$, what is the size $\gamma^{ID}(G)$ of minimum identifying code?
Facts about identifying codes

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- Motivation: fault-detection in processors networks
- Main question:

  Given a graph $G$, what is the size $\gamma^{ID}(G)$ of minimum identifying code?

- Existence $\iff$ no twins in the graph:

  Twins: $N[u] = N[v]$
A difficult question...

**Identifying Code**: Given a twin-free graph $G$ and an integer $k$, is there an identifying code of size $k$ in $G$?

**Proposition** Charon, Hudry, Lobstein, 2001

**Identifying Code** is **NP-complete**.
A difficult question...

**Identifying Code**: Given a twin-free graph $G$ and an integer $k$, is there an identifying code of size $k$ in $G$?

**Proposition** Charon, Hudry, Lobstein, 2001

**Identifying Code** is NP-complete.

- Best polynomial approximation with logarithmic factor
- Polynomial for trees
Outline

1. Bounds and extremal graphs
2. Study in restricted classes of graphs
3. Identifying colorings
4. Some perspectives
Part I

Bounds and extremal graphs
Bounds

$|V|$ : number of vertices

\[
\log(|V| + 1) \leq \gamma^{ID}(G) \leq |V| - 1
\]
Bounds

$|V|$ : number of vertices

$$\log(|V| + 1) \leq \gamma^{ID}(G) \leq |V| - 1$$

- Tight example:

```
  a  b  c
ab abc bc
ac
```
Bounds

$|V| : \text{number of vertices}$

$$\log(|V| + 1) \leq \gamma^{ID}(G) \leq |V| - 1$$


- Tight example:

- Complete characterization by Moncel in 2006.
Bounds

$|V| : \text{number of vertices}$

$$\log(|V| + 1) \leq \gamma^{ID}(G) \leq |V| - 1$$

- Tight example:

![Diagram 1](image1.png)

- Complete characterization by Moncel in 2006.
- Complete characterization?
Some tight examples and a conjecture

Stars

Conjecture
Charbit, Charon, Cohen, Hudry, Lobstein, 2008
Some tight examples and a conjecture

Stars

Complete graphs minus maximal matching
Some tight examples and a conjecture

Stars

Complete graphs minus maximal matching

**Conjecture** Charbit, Charon, Cohen, Hudry, Lobstein, 2008

These are the only graphs with $\gamma^{ID} = |V| - 1$. 
Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

(1) Star $K_{1,n}$,

(1) $K_{1,6}$
Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

(1) Star $K_{1,n}$,
(2) Graphs $P_{2k}^{k-1}$,

(1) $K_{1,6}$

(2) $P_6^2$
Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

(1) Star $K_{1,n}$,
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(3) Join of several graphs in (2) and/or with some $K_2$'s,

(1) $K_{1,6}$

(2) $P_6^2$

(3) $P_4 \bowtie P_4$
Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

(1) Star $K_{1,n}$,
(2) Graphs $P^{k-1}_{2k}$,
(3) Join of several graphs in (2) and/or with some $\overline{K}_2$'s,

(1) $K_{1,6}$
(2) $P^2_6$
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(3) $P_4 \boxtimes P_4 \boxtimes K_2$
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(1) $K_{1,6}$
(2) $P_6^2$
(3) $P_4 \Join P_4 \Join \overline{K_2}$
Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

1. Star $K_{1,n}$,
2. Graphs $P_{2k}^{k-1}$,
3. Join of several graphs in (2) and/or with some $K_2$'s,
4. A graph in (2) or (3) with a universal vertex.

(1) $K_{1,6}$

(2) $P_6^2$

(3) $P_4 \otimes P_4 \otimes \overline{K_2}$

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Characterization of graphs with $\gamma^{ID}(G) = \vert V \vert - 1$

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Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

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**Theorem** Foucaud, Guerrini, Kovše, Naserasr, P., Valicov, 2011

Let $G$ be a connected twin-free graph.

$\gamma^{ID}(G) = |V| - 1 \iff G$ in (1), (2), (3) or (4)
Ideas of the proof

Theorem Foucaud, Guerrini, Kovše, Naserasr, P., Valicov, 2011

$$\gamma^{ID}(G) = |V| - 1 \iff G \text{ in (1), (2), (3) or (4)}$$

$$\iff$$ By induction
Ideas of the proof

\[ \gamma^{ID}(G) = |V| - 1 \iff G \text{ in (1), (2), (3) or (4)} \]

\(\Leftarrow\) By induction
\(\Rightarrow\) Let \(G\) be a minimal counter-example.

- There is \(u \in V\) s.t. \(G - u\) extremal.
- By minimality, \(G - u\) is in (1), (2), (3) or (4).
- We can construct an identifying code of size \(|V| - 2\) of \(G\), contradiction.
Consequence

Corollary

If $\gamma^{ID}(G) = |V| - 1$, $G$ has maximum degree $\Delta \geq |V| - 2$. 
Consequence

If $\gamma^{ID}(G) = |V| - 1$, $G$ has maximum degree $\Delta \geq |V| - 2$.

Corollary

Upper bound with the maximum degree $\Delta$?

Conjecture  Foucaud, Klasing, Kosowski, Raspaud, 2012

$\gamma^{ID}(G) \leq |V| - \frac{|V|}{\Delta} + O(1)$.
Part II

Study in a restricted class of graphs:

Line graphs
Identifying code in line graphs

Identifying code

Edge identifying code

Pendant edges

Twins

Identifying code

$$\gamma_{EID}(G) = \gamma_{ID}(L(G))$$
Identifying code in line graphs

\[ G \]

\[ \mathcal{L}(G) \]

Identifying code in line graphs

Identifying code \[ \gamma_{EID}(G) \] = \[ \gamma_{ID}(L(G)) \]
Identifying code in line graphs

Identifying code

\[ \gamma(G) = \gamma(L(G)) \]
Identifying code in line graphs

\[ \gamma_{EID}(G) = \gamma_{ID}(L(G)) \]
Identifying code in line graphs

$G \xrightarrow{\mathcal{L}} L(G)$

Identifying code
Identifying code in line graphs

$G$

$\mathcal{L}(G)$

Edge identifying code $\rightarrow$ Identifying code

$\text{Identifying code}$

$\gamma_{EID}(G) = \gamma_{ID}(L(G))$
Identifying code in line graphs

Edge identifying code

$\gamma^{EID}(G)$

Identifying code

$\gamma^{ID}(\mathcal{L}(G))$

Pendant edges

Twins
Edge-IDCode : Given $G$ pendant-free and $k$, $\gamma^{EID}(G) \leq k$?

**Theorem** Foucaud, Gravier, Naserasr, P., Valicov, 2012

**Edge-IDCode** is **NP-complete** even for planar subcubic bipartite graphs with large girth.

Reduction from **Planar** $(\leq 3, 3)$-SAT.
Still difficult

**Edge-IDCode**: Given $G$ pendant-free and $k$, $\gamma^{EID}(G) \leq k$?

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**Edge-IDCode** is **NP-complete** even for planar subcubic bipartite graphs with large girth.

Reduction from **Planar** $(\leq 3, 3)$-**SAT**.

**Corollary**

**Identifying Code** is **NP-complete** even for perfect planar 3-colorable line graphs with maximum degree 4.
Bounds using the number of vertices

**Proposition** Foucaud, Gravier, Naseerasr, P., Valicov, 2012

\[ \frac{1}{2} |V(G)| \leq \gamma^{EID}(G) \leq 2|V(G)| - 3 \]
Bounds using the number of vertices

**Proposition** Foucaud, Gravier, Naserasr, P., Valicov, 2012

\[
\frac{1}{2} |V(G)| \leq \gamma^{EID}(G) \leq 2|V(G)| - 3
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- **Lower Bound**: a code must cover \(\simeq\) half of vertices.
  \(\rightarrow\) Tight for hypercubes.
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- **Upper Bound**: a minimal code is 2-degenerate.
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18/42
Bounds using the number of vertices

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- **Upper Bound:** a minimal code is 2-degenerate.
  \( \rightarrow \) Tight only for \( K_4 \).
  \( \rightarrow \) Infinite family with \( \gamma^{EID}(G) = 2|V(G)| - 6 \):

\[ \ldots \]
Bounds using the number of vertices

**Proposition** Foucaud, Gravier, Naserasr, P., Valicov, 2012

\[
\frac{1}{2} |V(G)| \leq \gamma^{EID}(G) \leq 2|V(G)| - 3
\]

**Corollary**

\textit{Edge-IDCode} has a polynomial 4-approximation.

- Best polynomial approximation for identifying codes in \(\log(|V|)\).
  
  (Laifenbeld, Trachtenberg, Berger-Wolf, 2006 and Gravier, Klasing, Moncel, 2008)
Bounds using the number of edges

**Proposition** Foucaud, Gravier, Naserasr, P., Valicov, 2012

\[
\frac{3}{2\sqrt{2}} \sqrt{|E(G)|} \leq \gamma^{EID}(G) \leq |E(G)| - 1
\]

- Upper Bound: from identifying code
- Lower Bound: using the lower bound for vertices

→ Tight for:
Bounds using the number of edges

**Proposition** Foucaud, Gravier, Naserasr, P., Valicov, 2012

\[ \frac{3}{2\sqrt{2}} \sqrt{|E(G)|} \leq \gamma^{EID}(G) \leq |E(G)| - 1 \]

**Corollary**

If \( G \) is a line graph, \( \gamma^{ID}(G) \geq \Theta(\sqrt{|V|}) \)
Conclusion for line graphs

• Class of graph for which $\gamma^{ID}(G) \geq \Theta(\sqrt{|V|})$ (instead of $\Theta(\log(|V|))$).

• Defined by forbidden induced subgraphs:

• Is the lower bound still true with less restrictions? For other classes defined by forbidden induced subgraphs?
  → False for claw-free graphs.
  → True for interval graphs.
Part III

A variation of identifying code:
Identifying colorings of graphs
Some variations

- Locating-dominating codes
- Resolving sets
- \((r, \leq \ell)\)-identifying codes
- Weak and light codes
- Tolerant identifying codes
- Watching systems
- Discriminating codes
- Adaptative identifying codes
- Locating colorings
- ...

Some variations

• Locating-dominating codes
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• ...

One more:
Identifying coloring
Proper coloring of graphs

→ Two adjacent vertices have different colors.

\[ \chi(G) = 3 \]

Chromatic number \( \chi(G) \): minimum number of colors needed
Proper coloring of graphs - a lower bound

Clique number $\omega(G)$: max. number of vertices that induces a complete graph

$\omega(G) = 3$

For any graph $G$, $\chi(G) \geq \omega(G)$
Proper coloring of graphs - a lower bound

Clique number \( \omega(G) \): max. number of vertices that induces a complete graph

\[
\omega(G) = 4
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Proper coloring of graphs - a lower bound

Clique number $\omega(G)$: max. number of vertices that induces a complete graph

$$\omega(G) = 4$$

For any graph $G$, $\chi(G) \geq \omega(G)$
...that is not always reached

\[ \chi(C_5) = 3 \text{ but } \omega(C_5) = 2 \]
...that is not always reached

\[ \chi(C_5) = 3 \text{ but } \omega(C_5) = 2 \]
Perfect graphs

Perfect graph (1963): $G$ is perfect if $\omega(H) = \chi(H)$ for any induced subgraph $H$ of $G$

**Theorem** Strong Perfect Graph Theorem (Chudnovsky *et al.* 2002)

$G$ is perfect if and only if it has no induced odd cycle or complement of odd cycle with more than 4 vertices
A part of the big family of perfect graphs
Identification with colors

Identifying codes

Proper graph colorings
Identification with colors

Identifying codes

Proper graph colorings

Identifying colorings
Locally identifying coloring

- Proper vertex coloring $c : V \to \mathbb{N}$
- **local** identification by the colors in the neighborhood: $c(N[x])$

$$c(N[x]) \neq c(N[y]) \text{ for } xy \in E$$

- $\chi_{lid}(G)$: min. number of colors in a lid-coloring of $G$. 
An example: the path

\[ \chi_{lid}(P_k) \leq 4 \]

With 3 colors:

\[ \chi_{lid}(P_k) = 3 \text{ iff } k \text{ is odd.} \]
An example: the path

\[ \chi_{\text{lid}}(P_k) \leq 4 \]

With 3 colors:

\[ \chi_{\text{lid}}(P_k) = 3 \text{ iff } k \text{ is odd.} \]
An example: the path

1, 2  1, 2, 3  2, 3, 4  1, 3, 4  1, 2, 4  1, 2, 3  2, 3, 4  3, 4
An example: the path

$\chi_{lid}(P_k) \leq 4$
An example: the path

\[ \chi_{lid}(P_k) \leq 4 \]

With 3 colors:

\[ \chi_{lid}(P_k) = 3 \text{ iff } k \text{ is odd.} \]
Link with chromatic number

- A lid-coloring is a proper coloring: $\chi_{lid} \geq \chi$.
- No upper bound with $\chi$.
  $\rightarrow$ complete graph $K_k$ subdivided twice: $\chi_{lid} = k$, $\chi = 3$
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  → complete graph $K_k$ subdivided twice: $\chi_{lid} = k$, $\chi = 3$

- Not monotone: $\chi_{lid}(P_5) \leq \chi_{lid}(P_4)$
\( \chi_{lid} \) is not monotone at all
$\chi_{lid}$ is not monotone at all

$$\chi_{lid}(G) = 5 \preccurlyeq k = \chi_{lid}(G - u)$$
Study in perfect graphs

- Perfect
  - Permutation
  - Cograph
  - Interval
  - Chordal
  - Split
  - $k$-tree
  - Bipartite
  - $\mathcal{L}$ (bipartite)
  - Tree
Study in perfect graphs

**Perfect**
- Permutation
- Cograph
- Interval
- Split
- k-tree

**Chordal**
- Bipartite
- Tree

\(\mathcal{L}(\text{bipartite})\)
Bipartite graphs: the path

\[ \chi_{lid}(P_k) \leq 4 \]
Bipartite graphs are 4-lid-colorable

If \( G \) is bipartite, \( \chi_{\text{lid}}(G) \leq 4 \).
Bipartite graphs are 4-lid-colorable

If $G$ is bipartite, $\chi_{\text{lid}}(G) \leq 4$.

<table>
<thead>
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<th>$L_0$</th>
<th>$\rightarrow$</th>
<th>${1, 2}$</th>
</tr>
</thead>
<tbody>
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Bipartite graphs are 4-lid-colorable

If $G$ is bipartite, $\chi_{lid}(G) \leq 4$. 

<table>
<thead>
<tr>
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<th>Colors</th>
<th>Color Sets</th>
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Bipartite graphs

General bounds: $3 \leq \chi_{lid}(B) \leq 4$. 
Bipartite graphs

General bounds: $3 \leq \chi_{lid}(B) \leq 4$.

$\chi_{lid}(B) = 3$: 

![Graph](image-url)
Bipartite graphs

General bounds: $3 \leq \chi_{lid}(B) \leq 4$.

$\chi_{lid}(B) = 3$:  

$\chi_{lid}(B) = 4$:  

In general, 3-Lid-Coloring is NP-complete in bipartite graphs.
Bipartite graphs

General bounds: $3 \leq \chi_{lid}(B) \leq 4.$

\[ \chi_{lid}(B) = 3: \quad \chi_{lid}(B) = 4: \]

In general... \textbf{3-LID-COLORING} is NP-complete in bipartite graphs
Perfect graphs - results and conjecture

- **Perfect**
  - Not bounded by $\omega$
  - **Permutation**
  - **Cograph** \( \leq 2\omega \)
  - **Interval** \( \leq 2\omega \)
  - **Split** \( \leq 2\omega \)
  - **Chordal**
  - **Bipartite** \( \leq 2\omega \)
  - **$L$ (bipartite)**
  - **$k$-trees** \( \leq 2\omega \)
  - **Trees** \( \leq 2\omega \)

Any chordal graph $G$ has a lid-coloring with $2^\omega(G)$ colors.

Conjecture: Esperet, Gravier, Montassier, Ochem, P., 2012
Perfect graphs - results and conjecture

Perfect

Permutation

Chordal

Cograph $\leq 2\omega$

Interval $\leq 2\omega$

Split $\leq 2\omega$

$k$-trees $\leq 2\omega$

Bipartite $\leq 2\omega$

Cograph $\leq 2\omega$

Trees $\leq 2\omega$

Perfect

Not bounded by $\omega$

Conjecture

Esperet, Gravier, Montassier, Ochem, P., 2012

Any chordal graph $G$ has a lid-coloring with $2\omega(G)$ colors.
A good method for coloring

- Outerplanar graphs: $L_i = \text{union of paths}$, 5 colors $\rightarrow 4 \times 5 = 20$ colors
- Planar graphs: $L_i = \text{outerplanar}$, 20 colors and 16 more colors $\rightarrow 4 \times 20 \times 16 = 1280$ colors (Gonçalves, P., Pinlou, 2012)
- Same idea for $K_k$-minor free graphs (Gonçalves, P., Pinlou, 2012)
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Part IV

Perspectives
Open questions

• Bounds and extremal graphs
  \[ \text{Conjecture } \gamma^{ID}(G) \leq n - \frac{n}{O(\Delta)} \]

• Study in restricted classes of graphs
  \[ \text{Other classes with } \gamma^{ID}(G) \geq \Theta(\sqrt{|V|})? \]
  \[ \text{Better approximation for line graphs?} \]

• Identifying colorings
  \[ \text{Better bound for planar graphs (between 8 and 1280...)} \]
  \[ \text{Conjecture } \chi_{lid} \leq 2\omega \text{ for chordal graphs} \]

• Generalization to hypergraph
A new approach with integer linear programming?

Identifying code problem is equivalent to the following problem:

\[
\begin{align*}
\text{min} & \quad \sum_{u \in V} x_u \\
\text{s.t} & \quad \sum_{u \in N[v]} x_u \geq 1 \quad \forall v \in V \quad \text{(domination)} \\
& \quad \sum_{u \in N[v] \Delta N[v']} x_u \geq 1 \quad \forall v \neq v' \in V^2 \quad \text{(separation)} \\
& \quad x_u \in \{0, 1\}
\end{align*}
\]

→ Subproblem of hitting set, covering set problems
→ New lower bounds, approximations, polynomial algorithm?
MERCI.