Cognitive profile in elementary algebra: the PÉPITE test interface.

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Abstract: The work presented here is part of a multidisciplinary project, called the PÉPITE project. Its aim is to set up a system to diagnose student profiles starting from a multidimensional analysis grid to evaluate their competences in elementary algebra. This paper deals with the conception, the making and the evaluation of an interface in Computer Based Learning Environments. In it, we present the method adopted to set up a software which proposes tasks to the students and collects data for the diagnosis module.

We consider here the problems linked to the re-use of didactic know-how based on pencil and paper tasks and in particular how to transfer these tasks to a data processing environment. These problems are considered from the interface conceptor’s point of view. In an attempt to make this clear, we make a distinction between ergonomy problems connected with the functioning of the interface and problems related to the field of application.

Keywords: secondary education, mathematics, interdisciplinary, Human Computer Interface, formative evaluation.
1. **INTRODUCTION**

Studies on Human-Computer Interaction draw attention to the fact that the user of a technical device faces a double problem: first, transferring his knowledge of the task and second, learning how to use the system (SENACH, 1993). In Computer Based Learning Environments, these meaning transfer problems for the students are called by the didacticians « computational transposition » (BALACHEFF, 1994) (ARTIGUE, 1995): technical and physical limitations interfere with the student's knowledge both at the representation level and at the action level. These limitations modify the perception of the effects of the actions. In our work, we use pedagogical material previously intended for a pencil and paper environment. This paper deals with the problems encountered by the conceptors of Intelligent Tutoring System interface during the transfer from pencil and paper tasks to computerised tasks.

In this paper, we consider these problems referring to a software created as part of the PÉPITE project, which aims to describe the students’ functioning in algebra, in order to establish their cognitive profile. The emphasis is laid on the method of conception we adopted and the defining of evaluation criteria for the software. First, we present an analysis of the pencil and paper tasks to be transferred and the aims of the system (needs analysis and task analysis). Then, we present the conception of the computerised tasks focusing on what we want to observe in the students’ productions. Finally, we present the experimentation of the software by the users.

2. **THE BASIS OF THE PÉPITE PROJECT**

The aim of the PÉPITE project is to build a computerised environment able to «modelise» the reasoning process of the 15 year old students of French secondary schools (the year before the end of secondary school studies certificate) in elementary algebra at the beginning of the year. The LINGOT project which will follow the PÉPITE project will use this «modelisation» to give the students appropriate learning situations likely to help the evolution of their knowledge. The idea is to seek out, in the student's way of functioning, the «grains» of knowledge (in French, the «pépites») to use as a basis for building on new knowledge.

According to (BARON and VIVET, 1995), the general problem of automatic diagnosis in an ITS is to infer information about the learner’s model from what is noticed of his behaviour. This is what we try to do through an analysis and an interpretation of the data collected during the interaction. In the PÉPITE project, we base our work on a rigorous didactic and cognitive study which has been validated (GRUGEON, 1995). This study enables us to build cognitive profiles of students, applying a multidimensional analysis grid to the answers given to a series of pencil and paper exercises (the pencil and paper tasks). In this part, we describe the basis of the project: the didactical analysis and the pencil and paper diagnosis tools created by Brigitte GRUGEON. After, we present the general architecture of the PÉPITE project.

2.1. **THE DIDACTICAL ANALYSIS**

This research in the didactics of mathematics sets out from the hypothesis of knowledge building: the students have built up pieces of knowledge which may be different from the knowledge of reference. Consequently, the productions of the students present coherences and regularities which correspond to their personal knowledge. One of the results of this study is a tool enabling us to analyse the productions in order to find starting points to modify their knowledge. This tool (Cf. figure 1) combines a series of pencil and paper tasks with a

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1 Involving, in computer science, Martial VIVET, Élisabeth DELOZANNE, Pierre JACOBONI and Stéphanie JEAN from the LIUM (Laboratoire d’Informatique de l’Université du Maine) and, in didactics of mathematics, Michèle ARTIGUE and Brigitte GRUGEON from the DIDIREM laboratory of Paris VII.

2 PÉPITE stands for gold nugget and LINGOT for ingot.

3 This thesis also contains a study of institutional relations of the students to algebra which is outside matters here.
multidimensional analysis grid making it possible to interpret the students’ production in order to establish their profile in elementary algebra.

**The pencil and paper tasks**

Three types of pencil and paper tasks are proposed to the students during a test:

- **technical exercises** whose aim is to determine numeric calculating and formal manipulating procedure,
- **recognition exercises** whose aim is to determine how students identify and interpret algebraic expressions in an algebraic context or in connection with other contexts,
- **modelling exercises** whose aims are to ascertain whether the students use the expected algebraic type of treatment, how they translate problems in the algebraic context and how they use the tools adapted for solving the problems.

In this didactical work, the students’ answers to the exercises are analysed « by hand » by the teacher with the multidimensional analysis grid.

![Figure 1: The pencil and paper diagnosis tool.](image1)

![Figure 2: Architecture of the PÉPITE project.](image2)

**The multidimensional analysis grid**

This grid is made up of six components (GRUGEON, 1995):

- the **algebraic treatment** component gives us the opportunity to determine the algebraic competence of the student compared with types of expected treatment, in terms of success / failure,
- the **arithmetic / algebra relationship** component enables us to infer the meaning given by the students to the algebraic process and to compare it with the arithmetic process,
- the **operationality of formal manipulation of algebraic writings** component aims to study the way students deal with algebraic expressions,
- the **articulation between the different contexts** component allows us to identify preferred ways
of dealing with and moving between the different semiotic contexts,
- the role of algebra component is intended to describe the way the students deal with algebra,
- the rationality in algebra component makes it possible to identify the use of algebra as a tool for generalisation and for proof.

A set of criteria is associated to each of these analysis components. During the correction of the tests, for each answer given by the student the teachers give global values defined by the different criteria of the analysis grid. Some of the global values are specified by local values linked to the exercise.

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>CRITERION</th>
<th>GLOBAL VALUE</th>
<th>LOCAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>interaction between different contexts</td>
<td>type of conversion</td>
<td>identifiable</td>
<td>step by step separated writing</td>
</tr>
<tr>
<td>method of solving</td>
<td>arithmetic</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Two examples of students' production on a pencil and paper task.

In the example (Cf. figure 3), the conjurer exercise is particularly concerned with the « type of conversion » criterion for the « interaction between different contexts » component. This criterion has three possible global values: « correct », « identifiable » and « unidentifiable ». The « identifiable » global value is further defined in this exercise by the local values « global linear writing with brackets » and « step by step separated writing », which make it possible to define the type of conversion used in the particular context of this exercise more precisely. For this criterion, the diagnosis has given the following two students the same « step by step separated writing » local value, but the « method of solving » criterion has an « arithmetic » value for Mérième (Cf. figure 4) and an « algebraic » value for Sébastien.

The students’ profiles

Applying the analysis grid to the work produced by a student on the pencil and paper tasks gives a set of values for each student for each task. This result then needs important didactic analysis. The very precise description of the behaviour is too detailed to be used as it is by the teachers (or by a computer). A transversal analysis of the grid's results enables us to establish a
higher level description: the « cognitive profiles » of the students. These profiles can be used to understand and modify the student's way of functioning and thus act against student underachievement.

These profiles have three levels of description:
– a quantitative description of algebraic competence in terms of success rates for the technical exercises and the modelling exercises (Cf. first part of figure 5),
– a qualitative description of functioning coherences, component by component, in terms of functioning modalities obtained by cross checking of values of some criteria on the whole set of exercises (Cf. second part of figure 5),
– a description of flexibility between contexts, represented by a diagram (Cf. figure 6).

<table>
<thead>
<tr>
<th></th>
<th>technical exercises</th>
<th>modelling exercises</th>
<th>recognition exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebraic treatment</td>
<td>40%</td>
<td>42%</td>
<td></td>
</tr>
<tr>
<td>arithmetic/algebraic relation</td>
<td></td>
<td></td>
<td>:= announcement of the solution</td>
</tr>
<tr>
<td>operationality of formal manipulation of algebraic writings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>role of algebra</td>
<td></td>
<td></td>
<td>no function</td>
</tr>
<tr>
<td>rationality of algebra</td>
<td></td>
<td></td>
<td>pragmatical proof</td>
</tr>
</tbody>
</table>

Figure 5: An extract from Mérième's profile, success rate and functioning modalities

![Diagram of flexibility between contexts](image)

Figure 6: An extract from Mérième's profile, diagram of flexibility between contexts

This diagnosis tool based on pencil and paper tasks has been tested several times. It has in particular been tested in June 1996 on 600 students of a third form class (14 year old students).

### 2.2. THE DIAGNOSIS IN PÉPITE

The PÉPITE project aims to automate the pencil and paper diagnosis tool invented by Brigitte GRUGEON. The architecture of PÉPITÉ contains three modules (Cf. figure 2, § 2.1):
– PÉPITEST which offers the students an adaptation of the pencil and paper tasks to the computer and which collects their answers. The difficulty of creating PÉPITEST resides in the transfer of the pencil and paper tasks to the computer platform. This software has now been completed and its creation is the work which will be presented and discussed in this paper.
– PÉPIDIAG which interprets and codes the students’ productions, from the data furnished by
Pépitem. It uses the multidimensional analysis grid and defines values given to criteria tested by the exercise.

- PépiProfil which, from the preceding codes, establishes the students’ profiles and presents them to the users (teachers or researchers). This last module has been developed but is not detailed here.

3. CONCEPTION OF PÉPITEM

In this paragraph, we present the conception procedure used to perfect Pépitem. We specify the dimensions, objectives and evaluation methods of the software. Then, we separate the ergonomic problems of using the system, from those linked to the transfer of the tasks from the pencil and paper environment to the data processing environment.

3.1. CONCEPTION PROCESS

Many books about the Human-Computer Interface ergonomy recommend that evaluation should be considered as a « state of mind » (Kolski, 1993) which must express itself throughout the design of a system. This preoccupation with validating design choices and detecting problems of use as early as possible often leads to the adoption of an iterative design process based on the making of prototypes that are tested and then modified if necessary. Using prototypes enables us to meet the aims of a user-centered design: creating a system which is easy to learn and to use (Preece et al. 1994). Another advantage of using prototypes is to ease communication with customers and within the multidisciplinary conception team (Senach, 1993), (Krief, 1992), (Kolski, 1993), (Van-Heyleen and Hiraclides, 1996) for example.

This process necessitates an early examination of the evaluation criteria and methods. Senach distinguishes two principal aspects for the evaluation, the utility of the product and its usability (Senach, 1993). The utility deals with the adequacy of the software to the high level objectives of the customer. The usability concerns the capacity of the software to allow the user to reach his objectives easily.

As far as Pépitem is concerned, the « user » is the student whose objective is to solve the exercises. The « customer » is the person or the system in charge of carrying out the diagnosis on the student’s productions. The usability of the software concerns the quality of the interface with regard to the ergonomic recommendations (Bastien and Scapin, 1993). Utility concerns the capacity of the software to take the student’s behaviour into account in order to establish the diagnosis. From the conceptor’s point of view, the problem lies with defining machine tasks that give equivalent data to the pencil and paper ones. The evaluation consists of specifying this equivalence.

The evaluation methods that we have selected for the usability dimension are the classical methods used for the Human-Computer Interfaces design. For the utility dimension, we rely on didactics of mathematics methods (Robert, 1992) (Artigue, 1990). During conception, the validation consists of checking by case-studies that the students’ productions on the Pépitem tasks allow the didacticians to apply the analysis grid to draw up the students’ cognitive profiles by hand.

3.2. USABILITY OF PÉPITEM

The quality of a Human-Computer Interface is not guaranteed by the simple fact that it meets certain style guidelines or respects a certain set of recommendations in the best possible way (Senach, 1993). Nevertheless ergonomic criteria gather experience in these domains and constitute a guide for conceptors. We discuss their being taken into account in Pépitem, while

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4 This distinction is introduced for the convenience of the presentation, the ergonomy is also concerned with the problems of tasks and not only with the superficial aspect of the interface.
referring to (BASTIEN and SCAPIN, 1993), but we must not forget the specificity of PÉPITEST, as an assessment environment.

Guidance is particularly important in the case of PÉPITEST. Each student will do the test only once, so there cannot be an initial contact session with the software. The student is guided by the instructions, by the structure of the screen, by modifications of the aspect of the cursor and by help-balloons.

The respect of the limitation of the workload criterion (concision, minimal actions and information density) is a problem in our context. Some functionalities of the software set up to facilitate use (for example the drag and drop) can draw the attention of some students away from mathematical activities. This diversion onto aspects of interface manipulation can disturb the diagnosis.

The objective being diagnosis and not learning, the student has an explicit control over the system. As in the pencil and paper environment the student can change exercises and modify his answers when he wishes. He can also skip questions and go back to them later.

The adaptability criterion is taken into account on the one hand by flexibility and on the other hand by the experience of the user. In PÉPITEST, the student can answer the same question in different ways (for example by using, or not using, a graphic tool). For the student who has previous experience with computers, PÉPITEST proposes classical tools such as cut - paste.

Given that the objective is diagnosis, no protection is possible concerning error management. However the student can always correct his entries.

Special attention was paid to the respect of coherence and homogeneity in the interface with regard to the setting out of information, the stability of the screen and the use of physical control devices.

The names and codes used in PÉPITEST are those imposed by the mathematics didacticians. The individual coding of the interface is reduced to the icons used in the buttons of the toolbar.

The criterion of compatibility concerns the degree of similarity between the various work environments of the user. It refers to the transfer problems that we treat in the next paragraph.

The validation consists of submitting our prototypes to the judgement of experts (experts in ergonomy and experts of the domain: didacticians and mathematics teachers), to informal tests by users, and finally to controlled experimentation in a high school class.

3.3. Utility of PÉPITEST

The activity of the student on the PÉPITEST tasks must furnish data allowing a didactician (and later software PÉPIDIAG) to apply the multidimensional grid of analysis.

In the conception phase of PÉPITEST, we retained the possibility for the didacticians to carry out the coding of the data obtained as a validation criterion. The profiles thus obtained could be confirmed by the teacher of the class. According to our validation criterion, we have on the one hand to create PÉPITEST tasks as close as possible to the pencil and paper tasks in order to obtain the « same » productions. On the other hand, we must take into account the computational transposition: the transfer of pencil and paper exercises and tools to a computational environment changes the test and has consequences on the students’ productions.

PÉPITEST is a set of 23 very different exercises: from closed questions (such as Multiple Choice Questions) to totally open questions (such as the conjurer exercise presented on figure 1). At first sight, pencil and paper tasks such as Multiple Choice Questions do not give any transfer problems. However, tasks that involve the production of sentences or of mathematical expressions (in particular the modelling exercises) give problems. We can fear that the users will simplify their syntax while typing sentences with the keyboard. With regard to mathematical expressions, different authors (including ARTIGUE, 1995) have noticed that the

5 POLLIER, quoted by BALBO (BALBO, 1994 p.101), considers that three experts are enough to detect 85% of the errors of this level.
transition for fractions or square roots from spatial writing (pencil and paper) to linear writing (the keyboard has only a fraction bar) distracts the students from mathematical tasks to low level tasks such as use of brackets.

We expect the experimentation to provide us with corpora to study to what extent these expression constraints modify the data and change the diagnosis. Indeed, each task gives special difficulties linked to the nature of the cognitive activities in play and specified in the analysis grid. The detailed study of each of these tasks is not our purpose here (JEAN, 1996). Here we will simply present an example.

3.4. AN EXAMPLE OF TRANSFER TASK

Complete the table by writing a sentence translating each step of the calculation program opposite the corresponding algebraic expression.

| step 1 | Let the initial number be \( x \) |
| step 2 | \( -x + 3 \) |
| step 3 | \( (-x + 3)^2 \) |
| step 4 | \( \frac{1}{(-x + 3)^2 + 4} \) |

Figure 7: A pencil and paper task example.

Figure 8: The same task in PépitEST.

This exercise (Cf. figure 7) tries to identify in the student's work the conversion rules used to pass from the algebraic context to the natural language context. In the pencil and paper corpus, we raised only two cases: either the students did not treat the question at all, or they used a very limited number of terms, of which we established the exhaustive list. We have chosen to transfer this exercise proposing a set of terms to construct the sentences (Cf. figure 8). The list of these terms is large enough to construct correct sentences, but also allows a number of errors, expected or not. This tool modifies the proposed activity: this is undoubtedly an aid provided to the students which does not however give any indication to the answer. The
exercise set up like this can help some students to begin their work without preventing them from giving erroneous answers.

At first sight, we expect more answers to this exercise in its PÉPITEST version than in its pencil and paper version.

4. EXPERIMENTATION

PÉPITEST was the object of an experimentation whose objectives were:

− to validate the interface from its usability point of view,
− to collect a corpus on the test software and to compare it to the pencil and paper corpus, in particular for the modelling exercises and some exercises for which the PÉPITEST version is very different from the pencil and paper version,
− to check by case-studies that the work produced by the students on the PÉPITEST tasks allow the didacticians to construct the students’ profiles « by hand ».

4.1. ORGANISATION OF THE EXPERIMENTATION

The experimentation took place in October 1996 in a fifth form class of thirty-two students in a high school in the suburbs of Paris. The test lasted 1H 45 on the usual timetable of the mathematics class. The students were put in two groups and each student had access to a computer.

These students were supposed to be accustomed to using a computer through technology classes at high school. It was impossible to organise a preliminary session with the software and of its tools.

We collected the traces of the session, a file that records the students’ answers to the exercises (equivalent to the information collected in the pencil and paper test) as well as another file logging information on the use of tools and the timing, a questionnaire filled in by each student, the sheets of notes taken by the observers during the experimentation (three didacticians, two computer scientists and the teacher of the class) and the rough copies of the students (if any).

4.2. RESULTS

We now present the results obtained concerning the three objectives that we fixed for this experimentation: usability of the interface, differences between the two types of corpora for the exercises and possibility of constructing the cognitive profiles from the students’ productions.

Usability of the software

After the first half-hour, the questions concerning the use of software disappeared. The questions asked by the students during this period concerned the use of a computer (keyboard, new paragraph, drag and drop, selection of fields), the use of PÉPITEST tools (the eraser), mathematics (use of brackets, calculation, terminology) and the mathematics with PÉPITEST (typing mathematical expressions). For the last two points, difficulties naturally lasted longer for some students.

Handling the software through solving the exercises therefore took less than half an hour. Globally, the guidance set up (structure of the screen, cursors, help balloons) worked well. We plan nevertheless to reduce the learning time by proposing two or three screens presenting the few basic manipulations necessary to the use of PÉPITEST.

Finally, we noticed, as other experimentations have already shown (DELOZANNE, 1994) (SCHNEIDERMAN, 1992) that the sophisticated tools invented by the conceptors are under-used.

Differences between the different corpora and « manual » building up of profiles

We noted, while observing the students during the test and while studying the traces of the
sessions, that the students behave globally in a similar manner with PÉPITEST and during the pencil and paper test: they did the test in order and did approximately the same number of exercises.

Some exercises are more attractive in PÉPITEST than in the pencil and paper test: actually the students did them easily. But others seem more boring in PÉPITEST because the students have to manipulate algebraic expressions in them, which is considerably harder with the computer. As we had foreseen, the typing of algebraic expressions gave problems to the students without however preventing them from writing them. Some exercises which were not much attempted in pencil and paper test are more attempted on PÉPITEST, in particular the exercise presented in figure 4. We did not note the opposite phenomenon.

Case-studies indicate that the productions on the modelling exercises are similar and that in the unusual exercises PÉPITEST accentuates the difficulties of certain students. This sheds more light on the type of behaviour that we are trying to identify.

Finally and especially, on the first analysed productions, the didactician was able to apply the analysis grid and to obtain cognitive profiles confirmed by the class teacher. This is undoubtedly the most interesting result from our point of view as conceptor.

5. CONCLUSION

In this paper, we have presented PÉPITEST software which collects data to diagnose the capacities of students in algebra. We have insisted on the conception problems and on evaluating the interface relying on work in Computer Based Learning Environments and in the ergonomy of interfaces. PÉPITEST has been completed and successfully underwent the test of experimentation in class. This concludes the first prototyping cycle for the iterative conception process (DELOZANNE, 1994) (VAN-HEYLEN and HIRACLIDES, 1996). In our conception method, we laid great importance on defining the evaluation criteria of PÉPITEST which means specifying the equivalence between the data from the pencil and paper tasks and those of PÉPITEST. Our validation criterion of PÉPITEST consists of verifying that the data obtained with the software permits us to build up profiles equivalent to the pencil and paper ones. The validation of the whole PÉPITE project consists of demonstrating that the profile built up by the machine with the data obtained by the software is equivalent to the profile built up by a didactician with the same data.

The first results presented here and in particular the creation of a corpus from PÉPITEST tasks, make it possible to seek for the didactic study necessary for the conception of PÉPITE’s module of diagnosis. Only this second phase will allow us, through case-studies and a statistic study, to validate the automatic diagnosis by comparing it to the manual diagnosis established from the pencil and paper corpus.

6. ACKNOWLEDGMENTS

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7. REFERENCES


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