

Accurate Curvature Estimation Along Digital Contours With Maximal Digital Circular Arcs

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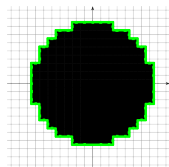
UMR 5127

Plan

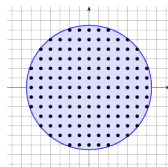
- 1 Introduction
- 2 Maximal digital circular arcs (MDCA)
- 3 Multigrid convergence
- 4 Experimental evaluation

Curvature estimation along digital contours

- Input : a digital contour C in the plane = topological border of a digital object
- Objective : associate a curvature field to C
- Difficulties : what is the correct curvature estimation ?



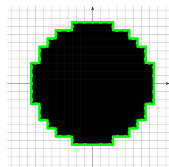
Input



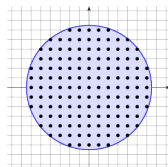
A disk ?

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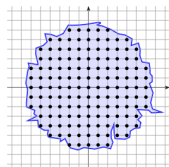


Input



A disk ?

?



also valid

State of the art

3 families [Worring,Smeulders93], [Vialard94], [Hermann,Klette07].

Curvature approached by :

1. derivative of the tangent orientation

often convolution by a Gaussian derivative kernel in a continuous [Worring,Smeulders93],[Vialard94] or discrete setting [Malgouyres,Brunet,Fourey09], [Fiorio,Mercat,Rieux10]

2. norm of the second derivative of the parameterized contour

local approximation with some polynomial [Marji03], [Hermann,Klette07]

3. inverse of radius of osculating circle

estimation of a local osculating circle [Coeurjolly,Miguet,Tougne01], [Hermann,Klette07], [Fleischmann,Wietzke,Sommer10]

State of the art (II)

Note

- these techniques require a user-given window or scale parameter
- hard to compare them objectively

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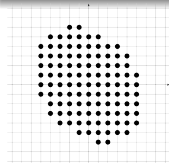
Two parameter-free methods

- Circumscribed circle to the two digital half-tangents
[\[Coeurjolly, Miguet, Tougne01\]](#)
Unstable in practice.
- **GMC** : Global Min-curvature estimator [\[Kerautret, Lachaud09\]](#)
Find the shape minimizing its squared-curvature while being digitized as the input contour.
Stable. Accurate. Can handle noisy data.

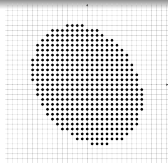
What is a good curvature estimator ?

Objective criterion

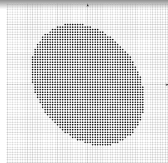
Asymptotic or Multigrid convergence When $h \rightarrow 0$ [Serra 82]



$\text{Dig}_X(h)$



$\text{Dig}_X(h/2)$



$\text{Dig}_X(h/4)$

...

...

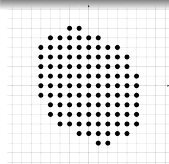
Geometric estimator $\hat{\epsilon}$ multigrid convergent for \mathcal{F} to a geom. quantity ϵ
 $\forall X \in \mathcal{F}, |\hat{\epsilon}(\text{Dig}_X(h)) - \epsilon(X)| \leq \tau(h)$, with $\lim_{h \rightarrow 0} \tau(h) = 0$.

- convergent estimators of area, perimeter, moments, tangents.

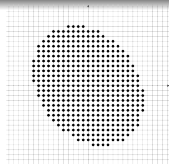
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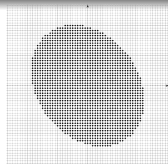
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- convergent estimators of area, perimeter, moments, tangents.
- **NO** convergent estimators of curvature

Contribution

We present a new curvature estimator to digital contours, based on the osculating circle (3rd family) :

- it is based on maximal digital circular arcs decomposition
- it requires no parameter
- it is multigrid convergent under some conditions
- it outperforms the best known curvature estimators in practice
- it is rather fast to compute (quasi-linear)

Contribution

We present a new curvature estimator to digital contours, based on the osculating circle (3rd family) :

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Outline

- definition of the curvature estimator based on **maximal digital circular arcs**
- **multigrid convergence** of the proposed estimator
- some experimental results

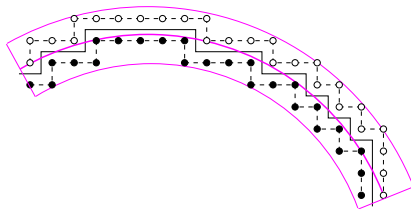
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Digital circular arc

Digital circular arc

A connected part C' of a contour C is a **Digital Circular Arc (DCA)** iff its interior points and exterior points are circularly separable.



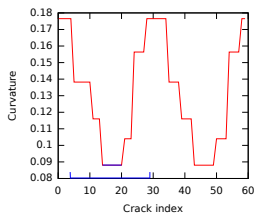
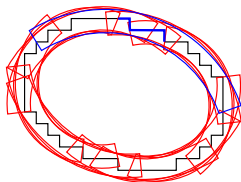
Curvature of a DCA A

Curvature of A $\kappa(A) = \begin{cases} 0 & \text{if } A \text{ linearly separable,} \\ \text{inverse radius of any separating circle.} \end{cases}$

Curvature estimator based on circular arcs

Maximal Digital circular arc (MDCA)

A DCA A is a MDCA iff all the proper supsets C' of A in the contour C ($A \subset C' \subset C$) are **not** DCA.



MDCA curvature estimator

Let $p \in C$, h the gridstep. Curvature $\hat{\kappa}^h(p) = \frac{1}{h}\kappa(A)$, where A is the most centered MDCA covering p .

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Definition of multigrid convergence

- Standard definition of multigrid convergence is for **global** geometric quantities
- Adaptation to local geometric quantities is delicate
- we follow the adaptation to tangent estimation [Lachaud, Vialard, de Vieilleville07]

Multigrid convergence of curvature estimator

The estimator $\hat{\kappa}$ is *multigrid-convergent* for the shapes \mathbb{X} if and only if, for any $X \in \mathbb{X}$, $h > 0$, for any $x \in \partial X$,

$$\begin{aligned} \forall y \in \partial \text{Dig}_h(X) \text{ with } \|y - x\|_1 \leq h, \\ |\hat{\kappa}^h(\text{Dig}_h(X), y) - \kappa(X, x)| \leq \tau_x(h), \end{aligned}$$

where $\tau_x : \mathbb{R}^{+*} \rightarrow \mathbb{R}^+$ has null limit at 0.

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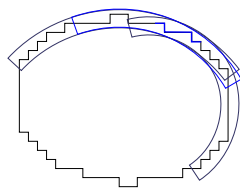
$$\forall y \in \partial \text{Dig}_h(X) \text{ with } \|y - x\|_1 \leq h, \quad (y \text{ close to } x)$$

$$|\hat{\kappa}^h(\text{Dig}_h(X), y) - \kappa(X, x)| \leq \tau_x(h), \quad (\text{implies } \hat{\kappa} \text{ close to } \kappa)$$

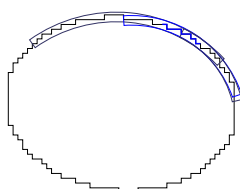
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Convergence theorem

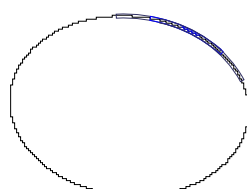
Let \mathbb{X} be the family of compact convex subsets of \mathbb{R}^2 , whose curvature field is continuous, strictly positive and upper bounded. If the **length of MDCA** along the digital contour of any $\text{Dig}_h(X)$, $X \in \mathbb{X}$, is lower bounded by $\Omega(h^a)$ and upper bounded by $O(h^b)$, $0 < b \leq a < 1/2$, then the curvature estimator $\hat{\kappa}^h$ is **uniformly multigrid convergent** for X , with $\tau = O(h^{\min(1-2a,b)})$.



$h = 1$



$h = 0.5$



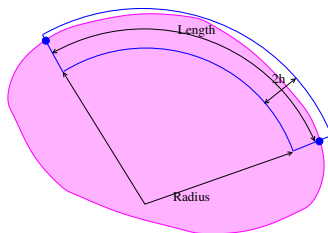
$h = 0.2$

Sketch of the proof

Let X be a convex shape, ∂X its boundary, $\text{Dig}_h(X)$ its digitization.

1. If a piece of ring \mathcal{R} of thickness $2h$ simply covers ∂X , and its Euclidean length is between $\Omega(h^a)$ and $O(h^b)$, then

$$\lim_{h \rightarrow 0} \text{radius}(\mathcal{R}) = 1/\kappa(p), \text{ for any } p \in \mathcal{R} \cap \partial X$$



Proof uses convex support functions.

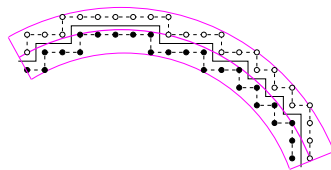
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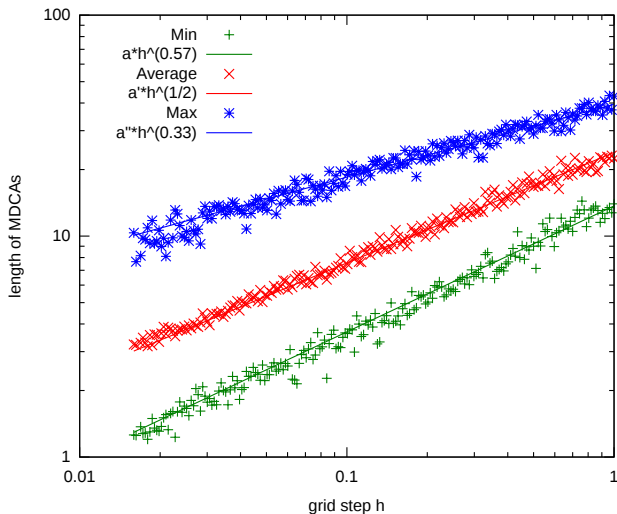
$$\lim_{h \rightarrow 0} \text{radius}(\mathcal{R}) = 1/\kappa(p), \text{ for any } p \in \mathcal{R} \cap \partial X$$

2. DCA are pieces of ring simply covering ∂X .

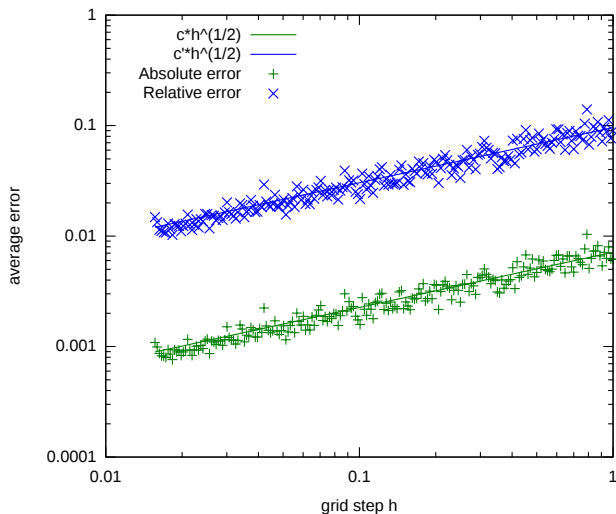


since $\partial \text{Dig}_h(X)$ has same topology as ∂X for small h (par-regularity).

Experimental evaluation of the length of MDCA



Experimental evaluation of the convergence



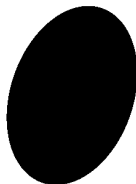
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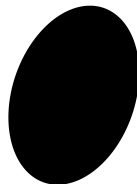
Curvature estimation on convex contours



$h = 1$

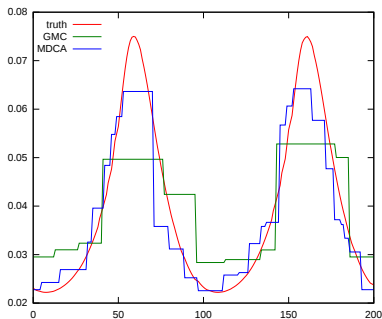


$h = 0.1$

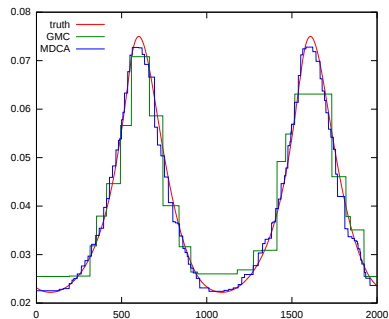


$h = 0.01$

Curvature estimation on convex contours

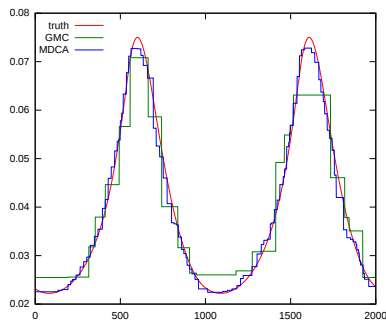


$h = 1$, GMC(1ms), MDCA(7ms)

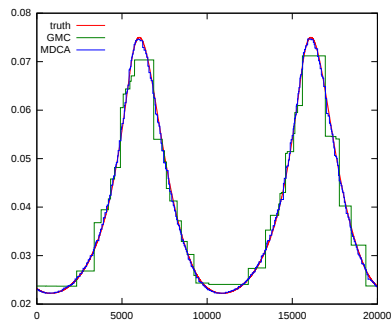


$h = 0.1$, GMC(27ms), MDCA(139ms)

Curvature estimation on convex contours

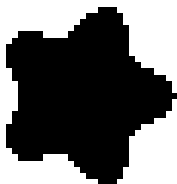


$h = 0.1$, GMC(27ms), MDCA(139ms)



$h = 0.01$, GMC(1843ms), MDCA(1783ms)

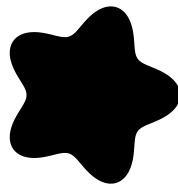
Curvature estimation on not convex contours



$h = 1$

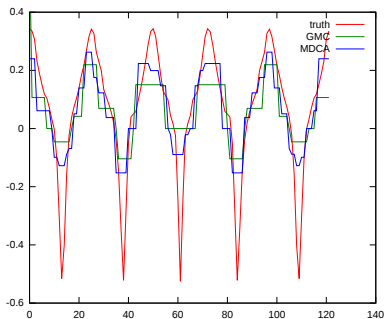


$h = 0.1$

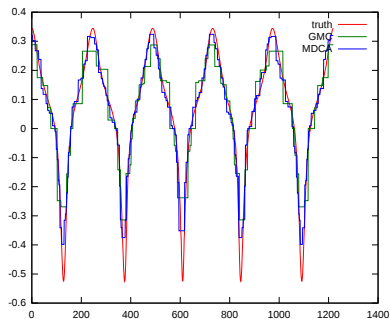


$h = 0.01$

Curvature estimation on not convex contours

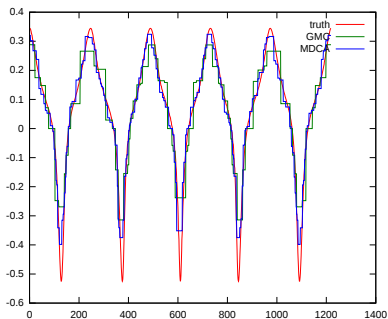


$h = 1$, GMC(1ms), MDCA(3ms)

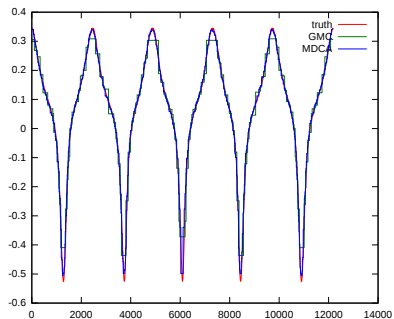


$h = 0.1$, GMC(15ms), MDCA(125ms)

Curvature estimation on not convex contours



$h = 0.1$, GMC(15ms), MDCA(127ms)



$h = 0.01$, GMC(843ms), MDCA(1195ms)

Discussion and perspectives

- New curvature estimator along digital contours based on maximal digital circular arcs
- + multigrid convergent if length of MDCA follows some asymptotic laws
- + experimentally multigrid convergent
- + more accurate than existing works
- + fast to compute (quasi-linear time, based on [\[Roussillon09\]](#))
- the asymptotic laws of the length of MDCAs remains to be proved
- for now not applicable to noisy digital contours