

Anytime discovery of a diverse set of patterns with Monte Carlo tree search

Guillaume Bosc¹ · Jean-François Boulicaut¹ ·
Chedy Raïssi² · Mehdi Kaytoue¹ 

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Abstract The discovery of patterns that accurately discriminate one class label from another remains a challenging data mining task. Subgroup discovery (SD) is one of the frameworks that enables to elicit such interesting patterns from labeled data. A question remains fairly open: How to select an accurate heuristic search technique when exhaustive enumeration of the pattern space is infeasible? Existing approaches make use of beam-search, sampling, and genetic algorithms for discovering a pattern set that is non-redundant and of high quality w.r.t. a pattern quality measure. We argue that such approaches produce pattern sets that lack of diversity: Only few patterns of high quality, and different enough, are discovered. Our main contribution is then to formally define pattern mining as a game and to solve it with Monte Carlo tree search (MCTS). It can be seen as an exhaustive search guided by random simulations which can be stopped early (limited budget) by virtue of its *best-first search* property. We show through a comprehensive set of experiments how MCTS enables the anytime discovery of a diverse pattern set of high quality. It outperforms other approaches when

Responsible editor: Johannes Fürnkranz.

✉ Mehdi Kaytoue
mehdi.kaytoue@insa-lyon.fr

Guillaume Bosc
guillaume.bosc@insa-lyon.fr

Jean-François Boulicaut
jean-francois.boulicaut@insa-lyon.fr

Chedy Raïssi
chedy.raïssi@inria.fr

¹ CNRS, INSA-Lyon, LIRIS, UMR5205, Université de Lyon, 69621 Lyon, France

² INRIA Nancy Grand Est, 54506 Villers-lès-Nancy, France

dealing with a large pattern search space and for different quality measures. Thanks to its genericity, our MCTS approach can be used for SD but also for many other pattern mining tasks.

Keywords Supervised pattern mining · Subgroup discovery · Exceptional model mining · Heuristic search · Monte Carlo tree search · Diversity

1 Introduction

The discovery of patterns, or descriptions, which discriminate a group of objects given a target (class label) has been widely studied as overviewed by Novak et al. (2009). Discovering such descriptive rules can be formalized as the so-called subgroup discovery task [SD introduced by Wrobel (1997)]. Given a set of objects, each being associated to a description and a class label, a subgroup is a description generalization whose discriminating ability is evaluated by a quality measure (F1-score, accuracy, etc). In the last two decades, different aspects of SD have been studied: The description and target languages (itemset, sequences, graphs on one side, quantitative and qualitative targets on the other), the algorithms that enable the discovery of the best subgroups, and the definition of measures that express pattern interestingness. These directions of work are closely related and many of the pioneer approaches were ad hoc solutions lacking from easy implementable generalizations [see for examples the surveys of Novak et al. (2009) and Duivesteijn et al. (2016)]. SD still faces two important challenges: First, how to characterize the interest of a pattern? Secondly, how to design an accurate heuristic search technique when exhaustive enumeration of the pattern space is unfeasible?

Leman et al. (2008) introduced a more general framework than SD called exceptional model mining (EMM). It tackles the first issue. EMM aims to find patterns that cover tuples that locally induce a model that substantially differs from the model of the whole dataset, this difference being measured with a quality measure. This rich framework extends the classical SD settings to multi-labeled data and it leads to a large class of models, quality measures, and applications (van Leeuwen and Knobbe 2012; Duivesteijn et al. 2016; Kaytoue et al. 2017). In a similar fashion to other pattern mining approaches, SD and EMM have to perform a heuristic search when exhaustive search fails. The most widely used techniques are *beam search* (van Leeuwen and Knobbe 2012; Meeng et al. 2014), *genetic algorithms* (del Jesús et al. 2007; Lucas et al. 2017), and *pattern sampling* (Moens and Boley 2014; Bendimerad et al. 2016).

The main goal of these heuristics is to drive the search towards the most interesting parts, i.e., the regions of the search space where patterns maximize a given quality measure. However, it often happens that the best patterns are *redundant*: They tend to represent the same description, almost the same set of objects, and consequently slightly differ on their pattern quality measures. Several solutions have been proposed to filter out redundant subgroups, e.g. as did Bringmann and Zimmermann (2009), van Leeuwen and Knobbe (2012), Meeng et al. (2014) and Bosc et al. (2016). Basically, a neighboring function enables to keep only local optima. However, one may end up

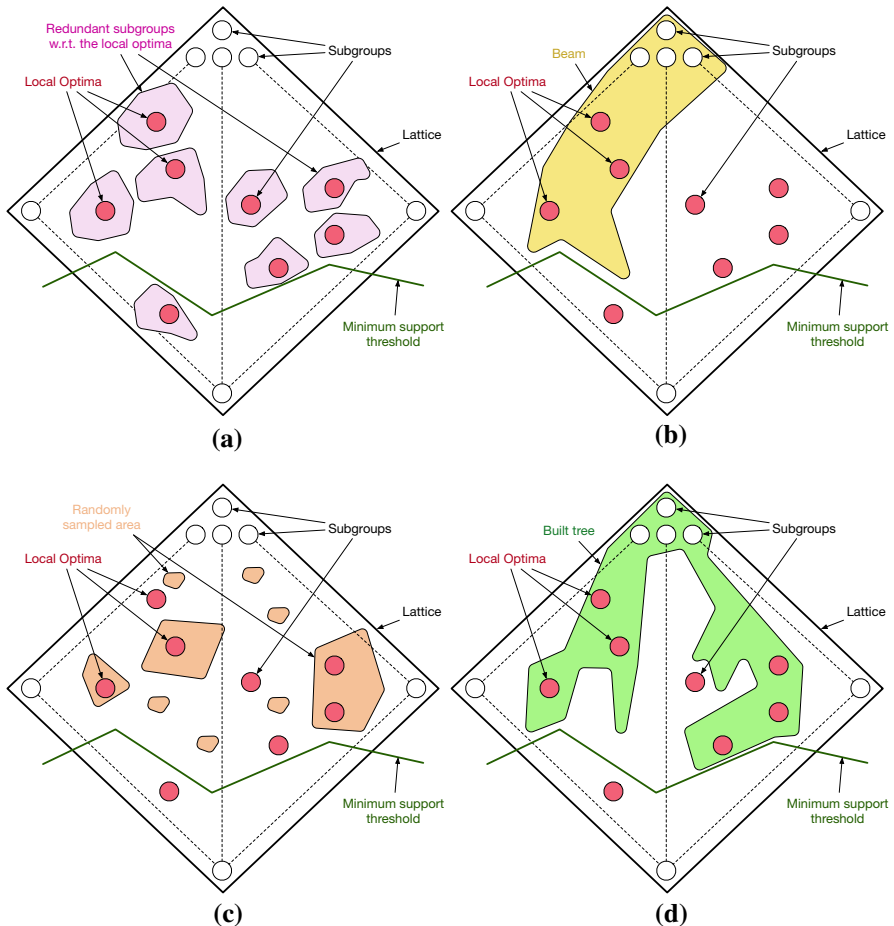


Fig. 1 Illustration of different SD search algorithms. **a** Redundancy problem. **b** Beam search. **c** Sampling exploration. **d** MCTS-based exploration

with a pattern set of small cardinality: This is the problem of *diversity*, that is, many local optima have been missed.

Let us illustrate this problem on Fig. 1. The search space of patterns, which can be represented as a lattice, hides several local optima (patterns maximizing a pattern quality measure in a neighborhood). Figure 1a presents such optima with red dots, surrounded with redundant patterns in their neighborhood. Given the minimal number of objects a pattern must cover, exhaustive search algorithms, such as SD-Map (Atzmüller and Puppe 2006; Atzmüller and Lemmerich 2009), are able to traverse this search space efficiently: The monotonicity of the minimum support and upper bounds on some quality measures such as the *weighted relative accuracy* (*WRAcc*) enable efficient and safe pruning of the search space. However, when the search space of patterns becomes tremendously large, either the number of patterns explodes or the search is intractable. Figure 1b presents beam-search, probably the most popular

technique within the SD and EMM recent literature. It operates a top-down level-wise greedy exploration of the patterns with a controlled level width that penalizes diversity [although several enhancements to favor diversity have been devised (van Leeuwen and Knobbe 2012; van Leeuwen and Ukkonen 2013; Meeng et al. 2014)]. Genetic algorithms have been proposed as well (Rodríguez et al. 2012; Pachón et al. 2011; Carmona et al. 2010). They give however no guarantees that all local optima will be found and they have been designed for specific pattern languages and quality measures (Lucas et al. 2017). Finally, pattern sampling is attractive as it enables direct interactions with the user for using his/her preferences to drive the search (Boley et al. 2011; Moens and Boley 2014). Besides, with sampling methods, a result is available anytime. However, traditional sampling methods used for pattern mining need a given probability distribution over the pattern space which depends on both the data and the measure and may be costly to compute (Boley et al. 2011; Moens and Boley 2014). Each iteration is independent and draws a pattern given this probability distribution (Fig. 1c).

In this article, we propose to support subgroup discovery with a novel search method, Monte Carlo tree search (MCTS). It has been mainly used in AI for domains such as games and planning problems, that can be represented as trees of sequential decisions (Browne et al. 2012). It has been popularized as definitively successful for the game of Go in Silver et al. (2016). MCTS explores a search space by building a game tree in an incremental and asymmetric manner: The tree construction is driven by random simulations and an exploration/exploitation trade-off provided by the so called upper confidence bounds (UCB) (Kocsis and Szepesvári 2006). The construction can be stopped anytime, e.g., when a maximal budget is reached. As illustrated on Fig. 1d, our intuition for pattern mining is that MCTS searches for some local optima, and once found, the search can be redirected towards other local optima. This principle enables per se a diversity of the result set: Several high quality patterns covering different parts of the data set can be extracted. More importantly, the power of random search leads to *anytime mining*: A solution is always available, it improves with time and it converges to the optimal one if given enough time and memory budget. This is a *best-first search*. Given a reasonable time and memory budget, MCTS quickly drives the search towards a diverse pattern set of high quality. Interestingly, it can consider, in theory, any pattern quality measure and pattern language [in contrast to current sampling techniques as developed by Boley et al. (2011) and Moens and Boley (2014)].

Our main contribution is to a complete characterization of MCTS for subgroup discovery and pattern mining in general. Revisiting MCTS in such a setting is not simple and the definition of it requires smart new policies. We show through an extensive set of experiments that MCTS is a compelling solution for a pattern mining task and that it outperforms the state-of-the-art approaches (exhaustive search, beam search, genetic algorithm, pattern sampling) when dealing with large search space of numerical and nominal attributes and for different quality measures.

The rest of this article is organized as follows. Section 2 formally introduces the pattern set discovery problem. Section 3 then recalls the basic definitions of MCTS. We present our MCTS method, called MCTS4DM, in Sect. 4. After discussing the related

work in Sect. 5, we report on experiments for understanding how to configure a MCTS for pattern mining (Sect. 6) and how does MCTS compare to competitors (Sect. 7).

2 Pattern set discovery

There exists several formal pattern mining frameworks and we choose here subgroup discovery to illustrate our purpose. We provide some basic definitions and then formally define pattern set discovery.

Definition 1 (*Dataset* $\mathcal{D}(\mathcal{O}, \mathcal{A}, \mathcal{C}, \text{class})$) Let \mathcal{O} , \mathcal{A} and \mathcal{C} be respectively a set of objects, a set of attributes, and a set of class labels. The domain of an attribute $a \in \mathcal{A}$ is $\text{Dom}(a)$ where a is either nominal or numerical. The mapping $\text{class} : \mathcal{O} \mapsto \mathcal{C}$ associates each object to a unique class label.

A subgroup can be represented either by a description (the pattern) or by its coverage, also called its extent.

Definition 2 (*Subgroup*) The description of a subgroup, also called pattern, is given by $d = \langle f_1, \dots, f_{|\mathcal{A}|} \rangle$ where each f_i is a restriction on the value domain of the attribute $a_i \in \mathcal{A}$. A restriction for a nominal attribute a_i is a symbol $a_i = v$ with $v \in \text{Dom}(a_i)$. A restriction for a numerical¹ attribute a_i is an interval $[l, r]$ with $l, r \in \text{Dom}(a_i)$. The description d covers a set of objects called the extent of the subgroup, denoted $\text{ext}(d) \subseteq \mathcal{O}$. The support of a subgroup is the cardinality of its extent: $\text{supp}(d) = |\text{ext}(d)|$.

The subgroup search space is structured as a lattice.

Definition 3 (*Subgroup search space*) The set of all subgroups forms a lattice, denoted as the poset (\mathcal{S}, \preceq) . The top is the most general pattern, without restriction. Given any $s_1, s_2 \in \mathcal{S}$, we note $s_1 \prec s_2$ to denote that s_1 is strictly more specific, i.e. it contains more stringent restrictions.

It follows that $\text{ext}(s_1) \subseteq \text{ext}(s_2)$ when $s_1 \preceq s_2$.

The ability of a subgroup to discriminate a class label is evaluated by means of a quality measure. The weighted relative accuracy (WRAcc), introduced by Lavrac et al. (1999), is among the most popular measures for rule learning and subgroup discovery. Basically, WRAcc considers the precision of the subgroup w.r.t. to a class label relatively to the appearance probability of the label in the whole dataset. This difference is weighted with the support of the subgroup to avoid to consider small ones as interesting.

Definition 4 (*WRAcc*) Given a dataset $\mathcal{D}(\mathcal{O}, \mathcal{A}, \mathcal{C}, \text{class})$, the WRAcc of a subgroup d for a label $l \in \text{Dom}(\mathcal{C})$ is given by:

$$\text{WRAcc}(d, l) = \frac{\text{supp}(d)}{|\mathcal{O}|} \times (p_d^l - p^l)$$

¹ We consider the finite set of all intervals from the data, without greedy discretization. As shown later, better patterns can be found in that case, when using only MCTS on large datasets.

Table 1 Toy dataset

ID	a	b	c	$class(\cdot)$
1	150	21	11	l_1
2	128	29	9	l_2
3	136	24	10	l_2
4	152	23	11	l_3
5	151	27	12	l_2
6	142	27	10	l_1

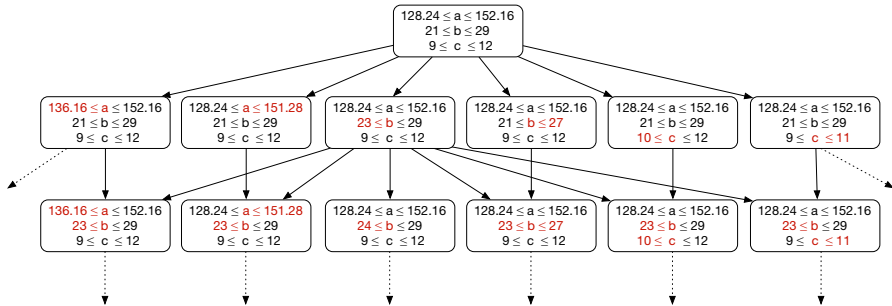


Fig. 2 The upper part of the search space for Table 1

where $p_d^l = \frac{|\{o \in ext(d) | class(o) = l\}|}{supp(d)}$ and $p^l = \frac{|\{o \in \mathcal{O} | class(o) = l\}|}{|\mathcal{O}|}$.

WRAcc returns values in $[-0.25, 0, 25]$, the higher and positive, the better the pattern discriminates the class label. Many quality measures other than WRAcc have been introduced in the literature of rule learning and subgroup discovery [Gini index, entropy, F score, Jaccard coefficient, etc. (Abudawood and Flach 2009)]. Exceptional model mining (EMM) considers multiple labels (label distribution difference in van Leeuwen and Knobbe (2012), Bayesian model difference in Duivesteijn et al. (2010), etc.). The choice of a pattern quality measure, denoted φ in what follows, is generally application dependent as explained by Fürnkranz et al. (2012).

Example 1 Consider the dataset in Table 1 with objects in $\mathcal{O} = \{1, \dots, 6\}$ and attributes in $\mathcal{A} = \{a, b, c\}$. Each object is labeled with a class label from $\mathcal{C} = \{l_1, l_2, l_3\}$. Consider an arbitrary subgroup with description $d = \langle [128 \leq a \leq 151], [23 \leq b \leq 29] \rangle$. Note that, for readability, we omit restrictions satisfied by all objects, e.g., $[9 \leq c \leq 12]$, and thus we denote that $ext(\langle \rangle) = \mathcal{O}$. The extent of d is composed of the objects in $ext(d) = \{2, 3, 5, 6\}$ and we have $WRAcc(d, l_2) = \frac{4}{6}(\frac{3}{4} - \frac{1}{2}) = \frac{1}{6}$. The upper part of the search space (most general subgroups) is given in Fig. 2. The direct specializations of a subgroup are given, for each attribute, by adding a restriction: Either by shrinking the interval of values to the left (take the right next value in its domain) or to the right (take the left next value). In this way, the finite set of all intervals taking borders in the attributes domain will be explored [see Kaytoue et al. (2011)].

Pattern set discovery consists in searching for a set of patterns $\mathcal{R} \subseteq \mathcal{S}$ of high quality on the quality measure φ and whose patterns are not redundant. As similar patterns

generally have similar values on φ , we design the pattern set discovery problem as the identification of the local optima w.r.t. φ . As explained below, this has two main advantages: Redundant patterns of lower quality on φ are pruned and the extracted local optima are diverse and potentially interesting patterns.

Definition 5 (*Local optimum as a non redundant pattern*) Let $sim : \mathcal{S} \times \mathcal{S} \rightarrow [0, 1]$ be a similarity measure on \mathcal{S} that, given a real value $\Theta > 0$, defines neighborhoods on $\mathcal{R} \subseteq \mathcal{S} : N_{\mathcal{R}}(x) = \{s \in \mathcal{R} \mid sim(x, s) \geq \Theta\}$. r^* is a local optimum of \mathcal{R} on φ iff $\forall r \in N_{\mathcal{R}}(r^*), \varphi(r^*) \geq \varphi(r)$. We denote by $filter(\mathcal{R})$ the set of local optima of \mathcal{R} and by $redundancy(\mathcal{R}) = 1 - \frac{|filter(\mathcal{R})|}{|\mathcal{R}|}$ the measure of redundancy of \mathcal{R} .

In this paper, the similarity measure on \mathcal{S} will be the Jaccard measure defined by

$$sim(r, r') = \frac{ext(r) \cap ext(r')}{ext(r) \cup ext(r')}$$

We propose to evaluate the diversity of a pattern set $\mathcal{R} \subseteq \mathcal{S}$ by the sum of the quality of its patterns. Indeed, the objective is to obtain the largest set of high quality patterns:

Definition 6 (*Pattern set diversity*) The diversity of a pattern set \mathcal{R} is evaluated by: $diversity(\mathcal{R}) = \sum_{r \in filter(\mathcal{R})} \varphi(r)$.

The function $filter()$ is generally defined in a greedy or heuristic way in the literature. van Leeuwen and Knobbe (2012) called it pattern set selection and we use their implementation in this article. First all extracted patterns are sorted according to the quality measure and the best one is kept. The next patterns in the order are discarded if they are too similar with the best pattern (Jaccard similarity between pattern supports is used). Once a non similar pattern is found, it is kept for the final result and the process is reiterated: Following patterns will be compared to it.

Problem 1 (*Pattern set discovery*) Compute a set of patterns $\mathcal{R}^* \subseteq \mathcal{S}$ such that $\forall r \in \mathcal{R}^*, r$ is a local optimum on φ and

$$\mathcal{R}^* = \operatorname{argmax}_{\mathcal{R} \subseteq \mathcal{S}} diversity(\mathcal{R}).$$

By construction, \mathcal{R}^* maximizes diversity and it minimizes redundancy. Naturally, \mathcal{R}^* is not unique. Existing approaches sometimes search for a pattern set of size k (Lucas et al. 2017), with a minimum support threshold $minSupp$ (Atzmüller and Puppe 2006).

3 Monte Carlo tree search

MCTS is a search method used in several domains to find an optimal decision [see the survey by Browne et al. (2012)]. It merges theoretical results from decision theory (Russell and Norvig 2010), game theory, Monte Carlo (Abramson 1990) and bandit-based methods (Auer et al. 2002). MCTS is a powerful search method because it enables the use of random simulations for characterizing a trade-off between the

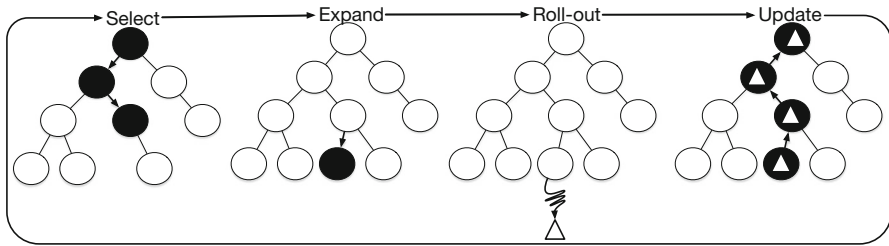


Fig. 3 One MCTS iteration. Taken from Browne et al. (2012)

exploration of the search tree and the exploitation of an interesting solution, based on past observations. Considering a two-players game (e.g., Go): The goal of MCTS is to find the best action to play given a current game state. MCTS proceeds in several (limited) iterations that build a partial game tree (called the search tree) depending on the results of previous iterations. The nodes represent game states. The root node is the current game state. The children of a node are the game states accessible from this node by playing an available action. The leaves are the terminal game states (game win/loss/tie). Each iteration, consisting of 4 steps (see Fig. 3), leads to the generation of a new node in the search tree (depending on the exploration/exploitation trade-off due to the past iterations) followed by a simulation (sequence of actions up to a terminal node). Any node s in the search tree is provided with two values: The number $N(s)$ of times it has been visited, and a value $Q(s)$ that corresponds to the aggregation of rewards of all simulations walked through s so far (e.g., the proportion of wins obtained for all simulations walked through s). The aggregated reward of each node is updated through the iterations such that it becomes more and more accurate. Once the computation budget is reached, MCTS returns the best move that leads to the child of the root node with the best aggregated reward $Q(\cdot)$.

In the following, we detail the 4 steps of a MCTS iteration applied to a game. Algorithm 1 gives the pseudo code of the most popular algorithm in the MCTS family, namely UCT (upper confidence bound for trees), as given in Kocsis and Szepesvári (2006).

The Select policy Starting from the root node, the SELECT method recursively selects an action (an edge) until the selected node is either a terminal game state or is not fully expanded (i.e., some children of this node are not yet expanded in the search tree). The selection of a child of a node s is based on the exploration/exploitation trade-off. For that, upper confidence bounds (UCB) are used. They bound the regret of choosing a non-optimal child. The original UCBs used in MCTS are the UCB1 from Auer et al. (2002) and the UCT from Kocsis and Szepesvári (2006):

$$UCT(s, s') = Q(s') + 2C_p \sqrt{\frac{2 \ln N(s)}{N(s')}}$$

where s' is a child of a node s and $C_p > 0$ is a constant (generally, $C_p = \frac{1}{\sqrt{2}}$). This step selects the most urgent node to be expanded, called s_{sel} in the following, considering

Algorithm 1 UCT: The popular MCTS algorithm.

```

1: function MCTS(budget)
2:   create root node  $s_0$  for current state
3:   while within computational budget budget do
4:      $s_{sel} \leftarrow \text{SELECT}(s_0)$ 
5:      $s_{exp} \leftarrow \text{EXPAND}(s_{sel})$ 
6:      $\Delta \leftarrow \text{ROLLOUT}(s_{exp})$ 
7:      $\text{UPDATE}(s_{exp}, \Delta)$ 
8:   end while
9:   return the action that reaches the child  $s$  of  $s_0$  with the highest  $Q(s)$ 
10: end function

11: function SELECT( $s$ )
12:   while  $s$  is non-terminal do
13:     if  $s$  is not fully expanded then return  $s$ 
14:     else  $s \leftarrow \text{BESTCHILD}(s)$ 
15:     end if
16:   end while
17:   return  $s$ 
18: end function

19: function EXPAND( $s_{sel}$ )
20:   randomly choose  $s_{exp}$  from non expanded children of  $s_{sel}$ 
21:   add new child  $s_{exp}$  to  $s_{sel}$ 
22:   return  $s_{exp}$ 
23: end function

24: function ROLLOUT( $s$ )
25:    $\Delta \leftarrow 0$ 
26:   while  $s$  is non-terminal do
27:     choose randomly a child  $s'$  of  $s$ 
28:      $s \leftarrow s'$ 
29:   end while
30:   return the reward of the terminal state  $s$ 
31: end function

32: function UPDATE( $s, \Delta$ )
33:   while  $s$  is not null do
34:      $Q(s) \leftarrow \frac{N(s) \times Q(s) + \Delta}{N(s) + 1}$ 
35:      $N(s) \leftarrow N(s) + 1$ 
36:      $s \leftarrow \text{parent of } s$ 
37:   end while
38: end function

39: function BESTCHILD( $s$ )
40:   return  $\arg \max_{s' \in \text{children of } s} UCB(s, s')$ 
41: end function

```

both the exploitation of interesting actions (given by the first term in UCT) and the exploration of lightly explored areas of the search space (given by the second term in UCT) based on the result of past iterations. The constant C_p can be adjusted to lower or increase the exploration weight in the exploration/exploitation trade-off. Note that when $C_p = \frac{1}{2}$, the UCT is called UCB1.

The Expand policy A new child, denoted s_{exp} , of the selected node s_{sel} is added to the tree according to the available actions. The child s_{exp} is randomly picked among all available children of s_{sel} not yet expanded in the search tree.

The RollOut policy From this expanded node s_{exp} , a simulation is played based on a specific policy. This simulation consists of exploring the search tree (playing a sequence of actions) from s_{exp} until a terminal state is reached. It returns the reward Δ of this terminal state: $\Delta = 1$ if the terminal state is a win, $\Delta = 0$ otherwise.

The Update policy The reward Δ is back-propagated to the root, updating for each parent the number of visits $N(\cdot)$ (incremented by 1) and the aggregation reward $Q(\cdot)$ (the new proportion of wins).

Example Figure 3 depicts a MCTS iteration. Each node has no more than 2 children. In this scenario, the search tree is already expanded: We consider the 9th iteration since 8 nodes of the tree have been already added. The first step consists in running the SELECT method starting from the root node. Based on a UCB, the *selection policy* chooses the left child of the root. As this node is fully expanded, the algorithm randomly selects a new node among the children of this node: Its right child. This selected node s_{sel} is not fully expanded since its left hand side child is not in the search tree yet. From this not fully expanded node s_{sel} , the EXPAND method adds the left hand side child s_{exp} of the selected node s_{sel} to expand the search tree. From this added node s_{exp} , a random simulation is rolled out until reaching a terminal state. The reward Δ of the terminal node is back-propagated with UPDATE.

4 Pattern set discovery with MCTS

Designing a MCTS approach for a pattern mining problem is different than for a combinatorial game: The goal is not to decide, at each turn, what is the best action to play, but to explore the search space: The pattern mining problem can thus be considered as a *single-turn single-player game*. Most importantly, MCTS offers a natural way to explore the search space of patterns with the benefit of the exploitation/exploration trade-off to improve diversity while limiting redundancy. For example, an exhaustive search will maximize diversity, but it will return a very large and redundant collection (but an exhaustive search is usually impossible). In contrast, a beam search can extract a limited number of patterns but it will certainly lack diversity (empirical evidences are given later in Sect. 7).

Before going into the formalization, let us illustrate how MCTS is applied to the pattern set discovery problem with Fig. 4. We consider here itemset patterns for the sake of simplicity, that is, subgroups whose descriptions are sets of items. We present an iteration of a MCTS for a transaction database with items $\mathcal{I} = \{a, b, c\}$. The pattern search space is given by the lattice $\mathcal{S} = (2^{\mathcal{I}}, \subseteq)$. The MCTS tree is built in a top-down fashion on this theoretical search space: The initial pattern, or root of the tree, is the empty set \emptyset . Assume that pattern $s_{sel} = \{a\}$ has been chosen by the *select* policy. During the *expand*, one of its direct specializations in $\{\{a, b\}, \{a, c\}\}$ is randomly chosen and added to the tree, e.g., $s_{exp} = \{a, c\}$. During the roll out,

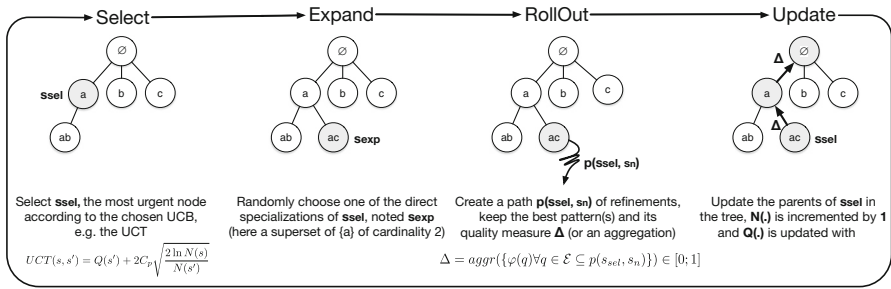


Fig. 4 A simple instantiation of MCTS for pattern mining

a simulation is run from this node: it generates a chain of specializations of s_{exp} called a path $p(s_{sel}, s_n)$ (a chain is a set of comparable patterns w.r.t. \subseteq , or \leq in the general case). The quality measure φ is computed for each pattern of the path, and an aggregated value (max, mean, etc.) is returned and called Δ . Finally, all parents of s_{exp} are updated: Their visit count $N(\cdot)$ is incremented by one, while their quality estimation $Q(\cdot)$ is recomputed with Δ (back propagation). The new values of $N(\cdot)$ and $Q(\cdot)$ will directly impact the selection of the next iteration when computing the chosen UCB, and thus the desired exploration/exploitation trade off. When the budget is exceeded (or if the tree is fully expanded), all patterns are filtered with a chosen pattern set selection strategy ($filter(\cdot)$).

The expected shape of the MCTS tree after a high number of iterations is illustrated in Fig. 1d. It suggests a high diversity of the final pattern set if given enough budget (i.e., enough iterations). However, how to properly define each policy (select, expand, roll out and update), is not obvious. Table 2 sums up the different policies that we use or develop specifically for a pattern mining problem.

4.1 The SELECT method

The SELECT method has to select the most promising node s_{sel} in terms of the exploration vs. exploitation trade-off. For that, the well-known bounds like UCT or UCB1 can be used. However, more sophisticated bounds have been designed for single player games. The single-player MCTS (SP-MCTS), introduced by Schadd et al. (2008), adds a third term to the UCB to take into account the variance σ^2 of the rewards obtained by the child so far. SP-MCTS of a child s' of a node s is:

$$SP-MCTS(s, s') = Q(s') + C \sqrt{\frac{2 \ln N(s)}{N(s')}} + \sqrt{\sigma^2(s') + \frac{D}{N(s')}}$$

where the constant C is used to weight the exploration term (it is fixed to 0.5 in its original definition) and the term $\frac{D}{N(s')}$ inflates the standard deviation for infrequently visited children (D is also a constant). In this way, the reward of a node rarely visited is considered as less certain: It is still required to explore it to get a more precise estimate

Table 2 The different policies**Select**

Choose one of the following UCB:

UCB1 or **UCB1-Tuned** or **SP-MCTS** or **UCT**

Expand

direct-expand: Randomly choose the next direct expansion

gen-expand: Randomly choose the next direct expansion until it changes the extent

label-expand: Randomly choose the next direct expansion until it changes the true positives

Activate LO: Generate each pattern only once (lectic enumeration)

Activate PU: Patterns with the same support/true positive set point to the same node

RollOut

naive-roll-out: Generate a random path of direct specializations of random length.

direct-freq-roll-out: Generate a random path of frequent direct specializations.

large-freq-roll-out: Generate a random paths of undirect specializations (random jumps).

Memory

no-memory: No pattern found during the simulation is kept for the final result.

top-k-memory: Top-k patterns of a simulation are considered in memory.

all-memory: All patterns generated during the simulation are kept.

Update

max-update: Only the maximum φ found in a simulation is back propagated

mean-update: The average of all φ is back-propagated

top-k-mean-update: The average of the best k φ is back-propagated

of its variance. If the variance is still high, it means that the subspace from this node is not homogeneous w.r.t. the quality measure and further exploration is needed.

Also, Auer et al. (2002) designed *UCB1-Tuned* to reduce the impact of the exploration term of the original UCB1 by weighting it with either an approximation of the variance of the rewards obtained so far or the factor 1/4. UCB1-Tuned of a child s' of s is:

$$UCB1-Tuned(s, s') = Q(s') + \sqrt{\frac{\ln N(s)}{N(s')} \min\left(\frac{1}{4}, \sigma^2(s') + \sqrt{\frac{2 \ln N(s)}{N(s')}}\right)}$$

The only requirement the pattern quality measure φ must satisfy is, in case of UCT only, to take values in $[0, 1]$: φ can be normalized in this case.

4.2 The EXPAND method

The EXPAND step consists in adding a pattern specialization as a new node in the search tree. In the following, we present different refinement operators, and how to avoid duplicate nodes in the search tree.

4.2.1 The refinement operators

A simple way to expand the selected node s_{sel} is to choose uniformly an available attribute w.r.t. s_{sel} , that is to specialize s_{sel} into s_{exp} such that $s_{exp} \prec s_{sel}$: s_{exp} is a refinement of s_{sel} . It follows that $ext(s_{exp}) \subseteq ext(s_{sel})$, and obviously $supp(s_{exp}) \leq supp(s_{sel})$, known as the monotonicity property of the support.

Definition 7 (*Refinement operator*) A refinement operator is a function $ref : \mathcal{S} \rightarrow 2^{\mathcal{S}}$ that derives from a pattern s a set of more specific patterns $ref(s)$ such that:

- (i) $\forall s' \in ref(s), s' \prec s$
- (ii) $\forall s'_i, s'_j \in ref(s), i \neq j, s'_i \not\prec s'_j, s'_j \not\prec s'_i$

In other words, a refinement operator gives to any pattern s a set of its specializations, that are pairwise incomparable (an anti-chain). The *refine* operation can be implemented in various ways given the kind of patterns we are dealing with. Most importantly, it can return all the direct specializations only to ensure that the exploration will, if given enough budget, explore the whole search space of patterns. Furthermore, it is unnecessary to generate infrequent patterns.

Definition 8 (*Direct-refinement operator*) A direct refinement operator is a refinement operator $directRef : \mathcal{S} \rightarrow 2^{\mathcal{S}}$ that derives from a pattern s the set of direct more specific patterns s' such that:

- (i) $\forall s' \in directRef(s), s' \prec s$
- (ii) $\nexists s'' \in \mathcal{S}$ s.t. $s' \prec s'' \prec s$
- (iii) For any $s' \in directRef(s)$, s' is frequent, that is $supp(s') \geq minSupp$

The notion of direct refinement is well known in pattern mining. For instance, the only way to refine a nominal (resp. Boolean) attribute is to assign it a value of its domain (resp. the *true* value). Refining an itemset consists in adding a item, while refining a numerical attribute can be done in two ways: Applying the minimal left change (resp. right change), that is, increasing the lower bound of the interval to the next higher value in its domain (resp. decreasing the upper bound to the next lower) as explained by Kaytoue et al. (2011). We still use the term *restriction* to denote the operations that create a direct refinement of pattern.

Definition 9 (*The direct-expand strategy*) We define the *direct-expand* strategy as follows: From the selected node s_{sel} , we randomly pick a—not yet expanded—node s_{exp} from $directRef(s_{sel})$ and add it in the search tree.

As most quality measures φ used in SD and EMM are solely based on the extent of the patterns, considering only one pattern among all those having the same extent is enough. However, with the *direct-refinement operator*, a large number of tree nodes may have the same extent as their parent. This redundancy may bias the exploration and more iterations will be required. For that, we propose to use the notion of closed patterns and their generators.

Definition 10 (*Closed descriptions and their generators*) The equivalence class of a pattern s is given by $[s] = \{s' \in \mathcal{S} \mid ext(s) = ext(s')\}$. Each equivalence class has a unique smallest element w.r.t. $<$ that is called the closed pattern: s is said to be closed iff $\nexists s'$ such that $s' < s$ and $ext(s) = ext(s')$. The non-closed patterns are called generators.

Definition 11 (*Generator-refinement operator*) A generator refinement operator is a refinement operator $genRef : \mathcal{S} \rightarrow 2^{\mathcal{S}}$ that derives from a pattern s the set of more specific patterns s' such that, $\forall s' \in genRef(s)$:

- (i) $s' \notin [s]$ (different support)
- (ii) $\nexists s'' \in \mathcal{S} \setminus genRef(s)$ s.t. $s'' \notin [s]$, $s'' \notin [s']$, $s' < s'' < s$ (direct next equivalence class)
- (iii) s' is frequent, that is $supp(s') \geq minSupp$ (frequent)

Definition 12 (*The gen-expand strategy*) To avoid the exploration of patterns with the same extent in a branch of the tree, we define the *min-gen-expand* strategy as follows: From the selected node s_{sel} , we randomly pick a—not yet expanded—refined pattern from $genRef(s_{sel})$, called s_{exp} , and add it to the search tree.

Finally, when facing a SD problem whose aim is to characterize a label $l \in \mathcal{C}$ we can adapt the previous refinement operator based on generators on the extents of both the subgroup and the label. As many other measures, the WRAcc seeks to optimize the (weighted relative) precision or accuracy of the subgroup. The accuracy is the ratio of true positives in the extent. We propose thus, for this kind of measures only, the *label-expand* strategy: Basically, the pattern is refined until the set of true positives in the extent changes. This minor improvement performs very well in practice (see Sect. 6).

4.2.2 Avoiding duplicates in the search tree

We define several refinement operators to avoid the redundancy within a branch of the tree, i.e., do not expand s_{sel} with a pattern whose extent is the same because the quality measure φ will be equal. However, another redundancy issue remains at the tree scale. Indeed, since the pattern search space is a lattice, a pattern can be generated in nodes from different branches of the Monte Carlo tree, that is, with different sequences of refinements, or simply permutations of refinements. As such, it will happen that a part of the search space is sampled several times in different branches of the tree. However, the visit count $N(s)$ of a node s will not count visits of other nodes that denote exactly the same pattern: The UCB is clearly biased. To tackle this aspect, we implement two methods: (i) Using a lexic order or (ii) detecting and unifying the duplicates within the tree. These two solutions can be used for any refinement operator. Note that enabling both these solutions at the same time is useless since each of them ensures to avoid duplicates within the tree.

Avoiding duplicates in the tree using a lexic order (LO)

Pattern enumeration without duplicates is at the core of constraint-based pattern-mining (Boulicaut and Jeudy 2010). Avoiding to generate patterns with the same extent

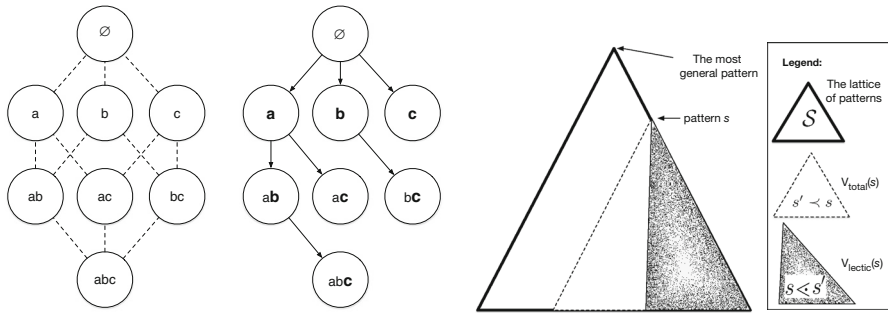


Fig. 5 Search space as a lattice (left), DFS of the search space (middle), and the principles of the normalized exploration rate

is usually based on a total order on the set of attribute restrictions. This poset is written by $(R, <)$.

Example 2 For instance, considering itemset patterns, $R = \mathcal{I}$ and a lectic order, usually the lexicographic order, is chosen on \mathcal{I} : $a < b < c < d$ for $I = \{a, b, c, d\}$ and $bc < ad$. Consider that a node s has been generated with a restriction r_i : we can expand the node only with restrictions r_j such that $r_i < r_j$. This total order also holds for numerical attributes by considering the minimal changes [see the work of Kaytoute et al. (2011) for further details].

We can use this technique to enumerate the lattice with a depth-first search (DFS), which ensures that each element of the search space is visited exactly once. An example is given in Fig. 5. However, it induces a strong bias: An MCTS algorithm would sample this tree instead of sampling the pattern search space. In other words, a small restriction w.r.t. $<$ has much less chances to be picked than a largest one. Going back to the example in Fig. 5 (middle), the item a can be drawn only once through a complete DFS; b twice; while c four times (in bold). It follows that patterns on the left hand side of the tree have less chances to be generated, e.g., $prob(\{a, b\}) = 1/6$ while $prob(\{b, c\}) = 1/3$. These two itemsets should however have the same chance to be picked as they have the same size. This variability is corrected by weighting the visit counts in the UCT with the normalized exploration rate [see Fig. 5 (right)].

Definition 13 (Normalized exploration rate) Let \mathcal{S} be the set of all possible patterns. The normalized exploration rate of a pattern s is,

$$\rho_{norm}(s) = \frac{V_{total}(s)}{V_{lectic}(s)} = \frac{|\{s' | s' \preceq s, \forall s' \in \mathcal{S}\}|}{|\{s' | (s \preceq s' \wedge s' \prec s) \vee s = s', \forall s' \in \mathcal{S}\}|}$$

Given this normalized exploration rate, we can adapt the UCBs when enabling the lectic order. For example, we can define the *DFS-UCT* of a child s' of a pattern s derived from the *UCT* as follows:

$$DFS-UCT(s, s') = Q(s') + 2C_p \sqrt{\frac{2 \ln(N(s) \cdot \rho_{norm}(s))}{N(s') \cdot \rho_{norm}(s')}}}$$

Proposition 1 (Normalized exploration rate for itemsets) *For itemsets, let s_i be the child of s obtained by playing action r_i and i is the rank of r_i in $(R, <)$: $\rho_{norm}(s_i) = \frac{2^{(|\mathcal{I}|-|s_i|)}}{2^{(|\mathcal{I}|-i-1)}}$.*

Proof Let $V_{lectic}(s_i)$ be the size of the search space sampled under s_i using a lectic enumeration, and $V_{total}(s_i)$ be the size of the search space without using a lectic enumeration. Noting $V_{total}(s_i) = 2^{(|\mathcal{I}|-|s_i|)}$ and $V_{lectic}(s_i) = 2^{(|\mathcal{I}|-i-1)}$ for itemsets, we have $\rho_{norm}(s_i) = \frac{V_{total}(s_i)}{V_{lectic}(s_i)} = \frac{2^{(|\mathcal{I}|-|s_i|)}}{2^{(|\mathcal{I}|-i-1)}}$. □

Proposition 2 (Normalized exploration rate for a numerical attribute) *For a single numerical attribute a , $\rho_{norm}(\cdot)$ is defined as follows :*

- Let $s' = \langle \alpha_i \leq a \leq \alpha_j \rangle$ obtained after a left change: $\rho_{norm}(s') = 1$.
- Let $s' = \langle \alpha_i \leq a \leq \alpha_j \rangle$ obtained after a right change. Let n be the number of values from $Dom(a)$ in $[\alpha_i, \alpha_j]$: $\rho_{norm}(s') = \frac{n+1}{2}$.

Proof As explained in the proof of (Proposition 1), $\rho_{norm}(s) = \frac{V_{total}(s)}{V_{lectic}(s)}$. For a numerical attribute, $V_{total}(s) = n(n+1)/2$, i.e. the number of all sub intervals. If s was obtained after a left change, $V_{lectic}(s) = n(n+1)/2$ as both left and right changes can be applied. If s was obtained after a right change, $V_{lectic}(s) = n$, as only n right changes can be applied. It follows that $\rho_{norm}(s) = \frac{n(n+1)/2}{n(n+1)/2} = 1$ if s was obtained from a left change and $\rho_{norm}(s) = \frac{n(n+1)/2}{n} = \frac{n+1}{2}$ otherwise. □

Avoiding duplicates in the tree using permutation unification (PU)

The permutation unification is a solution that enables to keep a unique node for all duplicates of a pattern that can be expanded within several branches of the tree. This is inspired from *Permutation AMAF* of Helmbold and Parker-Wood (2009), a method used in traditional MCTS algorithms to update all the nodes that can be concerned by a play-out. A unified node no longer has a single parent but a list of all duplicates' parent. This list will be used when back-propagating a reward.

This method is detailed in Algorithm 2. Consider that the node s_{exp} has been chosen as an expansion of the selected node s_{sel} . The tree generated so far is explored for finding s_{exp} elsewhere in the tree: If s_{exp} is not found, we proceed as usual; otherwise s_{exp} becomes a pointer to the duplicate node in the tree. In our MCTS implementation, we will simply use a hash map to store each pattern and the node in which is has been firstly encountered.

4.3 The ROLLOUT method

From the expanded node s_{exp} a simulation is run (ROLLOUT). With standard MCTS, a simulation is a random sequence of actions that leads to a terminal node: A game state from which a reward can be computed (win/loss). In our settings, it is not only the leaves that can be evaluated, but any pattern s encountered during the simulation. Thus, we propose to define the notion of path (the simulation) and reward computation (which nodes are evaluated and how these different rewards are aggregated) separately.

Algorithm 2 The permutation unification principle.

```

1:   $H \leftarrow$  new Hashmap()
2:  function EXPAND( $s_{sel}$ )
3:    randomly choose  $s_{exp}$  from non expanded children of  $s_{sel}$ 
4:    if ( $node \leftarrow H.get(s_{exp}) \neq null$ ) then
5:       $node.parents.add(s_{sel})$ 
6:       $s_{exp} \leftarrow node$ 
7:    else
8:       $s_{exp}.parents \leftarrow$  new List()
9:       $s_{exp}.parents.add(s_{sel})$ 
10:      $H.put(s_{exp}, s_{exp})$  ▷ A pointer on the unique occurrence of  $s_{exp}$ 
11:   end if
12:   add new child  $s_{exp}$  to  $s_{sel}$  in the tree ▷ Expand  $s_{sel}$  with  $s_{exp}$ 
13:   return  $s_{exp}$ 
14: end function

```

Definition 14 (*Path policy*) Let s_1 the node from which a simulation has to be run (i.e., $s_1 = s_{exp}$). Let $n \geq 1 \in \mathbb{N}$, we define a path $p(s_1, s_n) = \{s_1, \dots, s_n\}$ as a chain in the lattice $(\mathcal{S}, <)$, i.e., an ordered list of patterns starting from s_1 and ending with s_n such that $\forall i \in \{1, \dots, n-1\}$, s_{i+1} is a (not necessarily direct) refined pattern of s_i .

- *naive-roll-out*: a path of direct refinements is randomly created with length $pathLength \in \mathbb{N}^+$ a user-defined parameter.
- *direct-freq-roll-out*: The path is extended with a randomly chosen restriction until it meets an infrequent pattern s_{n+1} using the direct refinement operator. Pattern s_n is a leaf of the tree in our settings.
- *large-freq-roll-out*: overrides the *direct-freq-roll-out*: policy by using specializations that are not necessarily direct. Several actions are added instead of one to create a new element of the path. The number of added actions is randomly picked in $(1, \dots, jumpLength)$ where $jumpLength$ is given by the user ($jumpLength = 1$ gives the previous policy). This techniques allows to visit deep parts of the search space with shorter paths.

Definition 15 (*Reward aggregation policy*) Let s_1 be the node from which a simulation has been run and $p(s_1, s_n)$ the associated random path. Let $\mathcal{E} \subseteq p(s_1, s_n)$ be the subset of nodes to be evaluated. The aggregated reward of the simulation is given by: $\Delta = aggr(\{\varphi(s) \forall s \in \mathcal{E}\}) \in [0, 1]$ where $aggr$ is an aggregation function. We define several reward aggregation policies:

- *terminal-reward*: $\mathcal{E} = \{s_n\}$ and $aggr$ is the identity function.
- *random-reward*: $\mathcal{E} = \{s_i\}$ with a random $1 \leq i \leq n$ and $aggr$ is the identity function.
- *max-reward*: $\mathcal{E} = p(s_1, s_n)$ and $aggr$ is the $max(\cdot)$ function
- *mean-reward*: $\mathcal{E} = p(s_1, s_n)$ and $aggr$ is the $mean(\cdot)$ function.
- *top-k-mean-reward*: $\mathcal{E} = top-k(p(s_1, s_n))$, $aggr$ is the $mean(\cdot)$ function and $top-k(\cdot)$ returns the k elements with the highest φ .

A basic MCTS forgets any state encountered during a simulation. This is not optimal for single player games as relate Björnsson and Finnsson (2009): A pattern with a

high φ should not be forgotten as we might not expand the tree enough to reach it. We propose to consider several memory strategies.

Definition 16 (*Roll-out memory policy*) A roll-out memory policy specifies which of the nodes of the path $p = (s_1, s_n)$ shall be kept in an auxiliary data structure M .

- *no-memory*: Any pattern in \mathcal{E} is forgotten.
- *all-memory*: All evaluated patterns in \mathcal{E} are kept.
- *top-k-memory*: A list M stores the best k patterns in \mathcal{E} w.r.t. $\varphi(\cdot)$.

This structure M will be used to produce the final pattern set.

4.4 The UPDATE method

The backpropagation method updates the tree according to a simulation. Let s_{sel} be the selected node and s_{exp} its expansion from which the simulation is run: This step aims at updating the estimation $Q(\cdot)$ and the number of visits $N(\cdot)$ of each parent of s_{exp} recursively. Note that s_{exp} may have several parents when we enable permutation unification (PU). The number of visits is always incremented by one. We consider three ways of updating $Q(\cdot)$:

- *mean-update*: $Q(\cdot)$ is the average of the rewards Δ back-propagated through the node so far (basic MCTS).
- *max-update*: $Q(\cdot)$ is the maximum reward Δ back-propagated through the node so far. This strategy enables to identify a local optimum within a part of the search space that contains mostly of uninteresting patterns. Thus, it gives more chance for this area to be exploited in the next iterations.
- *top-k-mean-update*: $Q(\cdot)$ average of the k best rewards Δ back-propagated through the node so far. It gives a stronger impact for the parts of the search space containing several local optima.

mean-update is a standard in MCTS techniques. We introduce the *max-update* and *top-k-mean-update* policies as it may often happen that high-quality patterns are rare and scattered in the search space. The mean value of rewards from simulations would converge towards 0 (there are too many low quality subgroups), whereas the maximum value (and top-k average) of rewards enables to identify the promising parts of the search space.

4.5 Search end and result output

There are two ways a MCTS ends: Either the computational budget is reached (number of iterations) or the tree is fully expanded (an exhaustive search has been possible, basically when the size of the search space is smaller than the number of iterations). Indeed, the number of tree nodes equals the number of iterations that have been performed. It remains now to explore this tree and the data structure M built by the memory policy to output the list of diverse and non-redundant patterns.

Let $\mathcal{P} = T \cup M$ be a pool of patterns, where T is the set of patterns stored in the nodes of the tree. The set \mathcal{P} is totally sorted w.r.t. φ in a list \mathcal{A} . Thus, we have to

pick the k -best diverse and non-redundant subgroups within this large pool of nodes Λ to return the result set of subgroups $\mathcal{R} \subseteq \mathcal{P}$. For that, we choose to implement $filter(\cdot)$ in a greedy manner as done by van Leeuwen and Knobbe (2012) and Bosc et al. (2016). $\mathcal{R} = filter(\mathcal{P})$ as follows: A post-processing that filters out redundant subgroups from the diverse pool of patterns Λ based on the similarity measure sim and the maximum similarity threshold Θ . Recursively, we poll (and remove) the best subgroup s^* from Λ , and we add s^* to \mathcal{R} if it is not redundant with any subgroup in \mathcal{R} . It can be shown easily that $redundancy(\mathcal{R}) = 0$.

Applying $filter(\cdot)$ at the end of the search requires however that the pool of patterns \mathcal{P} has a reasonable cardinality which may be problematic with MCTS in term of memory. The allowed budget always enables such post-processing in our experiments (up to one million iterations).

5 Related work

SD aims at extracting subgroups of individuals for which the distribution on the target variable is statistically different from the whole (or the rest of the) population (Klösgen 1996; Wrobel 1997). Two similar notions have been formalized independently and then unified by Novak et al. (2009): Contrast set mining and emerging patterns. Close to SD, redescription mining aims to discover redescriptions of the same group of objects in different views (van Leeuwen and Galbrun 2015). Exceptional model mining (EMM) was first introduced by Leman et al. (2008) [see a comprehensive survey by Duivesteijn et al. (2016)]. EMM generalizes SD dealing with more complex target concepts: There are not necessarily one but several target variables to discriminate. EMM seeks to elicit patterns whose extents induce a model that substantially deviates from the one induced by the whole dataset.

First exploration methods that have been proposed for SD/EMM are exhaustive search ensuring that the best subgroups are found, e.g. Klösgen (1996), Wrobel (1997), Kavsek and Lavrac (2006) and Atzmüller and Lemmerich (2009). Several pruning strategies have been used to avoid the exploration of uninteresting parts of the search space. These pruning strategies are usually based on the monotonic (or anti-monotonic) property of the support or upper bounds on the quality measure (Grosskreutz et al. 2008; Kaytoue et al. 2017). To the best of our knowledge, the most efficient algorithms are (i) *SD-MAP** by Atzmüller and Lemmerich (2009) which is based on the FP-growth paradigm (Han et al. 2000) and (ii) an exhaustive exploration with optimistic estimates on different quality measures (Lemmerich et al. 2016). When an exhaustive search is not possible, heuristic search can be used. The most widely used techniques in SD and EMM are *beam search*, *evolutionary* algorithms and *sampling* methods. Beam search performs a level-wise exploration of the search space: A beam of a given size (or dynamic size for recent work) is built from the root of the search space. This beam only keeps the most promising subgroups to extend at each level (Lavrac et al. 2004; Mueller et al. 2009; van Leeuwen and Knobbe 2012). The redundancy issue due to the beam search is tackled with the pattern skyline paradigm by van Leeuwen and Ukkonen (2013), and with a ROC-based beam search variant for SD by Meeng et al. (2014). Another family of SD algorithms relies on evolutionary approaches. They use

a fitness function to select which individuals to keep at the next generated population. *SDIGA* (del Jesús et al. 2007) is based on a fuzzy rule induction system where a rule is a pattern in disjunctive normal form (DNF). Other approaches have been then proposed, generally ad-hoc solutions suited for specific pattern languages and selected quality measures (Rodríguez et al. 2012; Pachón et al. 2011; Carmona et al. 2010).

Finally, pattern sampling techniques are gaining interest. Moens and Boley (2014) employ controlled direct pattern sampling (CDPS). It enables to create random patterns with the help of a procedure based on a controlled distribution as did Boley et al. (2011). This idea was extended by Bendimerad et al. (2016) for a particular EMM problem to discover exceptional models induced by attributed graphs. Pattern sampling is attractive as it supports direct interactions with the user for using his/her preferences to drive the search. Besides, with sampling methods, a result is available anytime. However, traditional sampling methods used in pattern mining need a given probability distribution over the pattern space: This distribution depends on both the data and the measures (Boley et al. 2011; Moens and Boley 2014). Each iteration is independent and consists of drawing a pattern given this probability distribution. Moreover, these probability distributions exhibit the problem of the long tail: There are many more uninteresting patterns than interesting ones. Thus, the probability to draw an uninteresting pattern is still high, and not all local optima may be drawn: There are no guaranties on the diversity of the result set. Recently, the sampling algorithm MISERE has been proposed by Gay and Boullé (2012) and Egho et al. (2015, 2017). Contrary to the sampling method of Moens and Boley, MISERE does not require any probability distribution. It is agnostic of the quality measure but it still employs a discretization of numerical attribute in a pre-processing task. To draw a pattern, MISERE randomly picks an object in the data, and thus it randomly generalizes it into a pattern that is evaluated with the quality measure. Each draw is independent and thus the same pattern can be drawn several times. Finally, MCTS samples the search space without any assumption about the data and the measure. Contrary to sampling methods, it stores the result of the simulations of the previous iterations and it uses this knowledge for the next iterations: The probability distribution is learned incrementally. If given enough computation budget, the exploration/exploitation trade-off guides the exploration to all local optima (an exhaustive search). To the best of our knowledge, MCTS has never been used in pattern mining, however, Gaudel and Sebag (2010) designed the algorithm FUSE (Feature UCT Selection) which extends MCTS to a feature selection problem. This work aims at selecting the features from a feature space that are the more relevant w.r.t. the classification problem. For that, Gaudel and Sebag explore the powerset of the features (i.e., itemsets where the items are the features) with a MCTS method to find the sets of features that minimize the generalization error. Each node of the tree is a subset of feature, and each action consists of adding a new feature in the subset of features. The authors focus on reducing the high branching factor by using *UCB1-Tuned* and *RAVE* introduced by Gelly and Silver (2007). The latter enables to select a node even if it remains children to expand. The aim of FUSE is thus to return the best subset of features (the most visited path of the tree), or to rank the features with the RAVE score.

Table 3 Benchmark datasets experimented on in the SD and EMM literature

Name	# Objects	# Attributes	Type of attributes	Target attribute
<i>Bibtex</i>	7395	1836	Binary	TAG_statphys23
<i>BreastCancer</i>	699	9	Numeric	Benign
<i>Cal500</i>	502	68	Numeric	Angry-Aggressive
<i>Emotions</i>	594	72	Numeric	Amazed-surprised
<i>Ionosphere</i>	352	35	Numeric	Good
<i>Iris</i>	150	4	Numeric	Iris-setosa
<i>Mushroom</i>	8124	22	Nominal	Poisonous
<i>Nursery</i>	12,961	8	Nominal	class=priority
<i>Olfaction</i>	1689	82	Numeric	Musk
<i>TicTacToe</i>	958	9	Nominal	Positive
<i>Yeast</i>	2417	103	Numeric	Class1

6 Empirical evaluation on how to parameterize MCTS4DM

Our MCTS implementation for pattern mining, called MCTS4DM is publicly available.² As there are many ways to configure MCTS4DM, we propose first to study the influence of the parameters on runtime, pattern quality and diversity. We both consider benchmark and artificial data. The experiments were carried out on an Intel Core i7 CPU 4 Ghz machine with 16 GB RAM running under Windows 10.

6.1 Data

Firstly, we gathered benchmark datasets used in the recent literature of SD and EMM, that is, from van Leeuwen and Knobbe (2012), Downar and Duivesteijn (2017), van Leeuwen and Galbrun (2015), van Leeuwen and Knobbe (2011) and Duivesteijn and Knobbe (2011). Table 3 lists them, mainly taken from the UCI repository, and we provide some of their properties.

Secondly, we used a real world dataset from neuroscience. It concerns olfaction (see Table 3). This data provides a very large search space of numerical attributes [more details on the application are presented by Bosc et al. (2016)].

Finally, to be able to specifically evaluate diversity, a ground-truth is required. Therefore, we create an artificial data generator to produce datasets where patterns with a controlled WRAcc are hidden. The generator takes the parameters given in Table 4 and it works as follow. A data table with nominal attributes is generated with a binary target. The number of objects, attributes and attributes values are controlled with the parameters *nb_obj*, *nb_attr* and *domain_size*. Our goal is to hide *nb_patterns* patterns in noise: We generate random descriptions of random lengths $Ground = \{d_i \mid i \in [1, nb_patterns]\}$. For each pattern, we generate *pattern_sup* objects

² <https://github.com/guillaume-bosc/MCTS4DM>.

Table 4 Parameters of the artificial data generator

Name	Description	\mathcal{P}_{small}	\mathcal{P}_{medium}	\mathcal{P}_{large}
<i>nb_obj</i>	Number of objects	2000	20,000	50,000
<i>nb_attr</i>	Number of attributes	5	5	25
<i>domain_size</i>	Domain size per attribute	10	20	50
<i>nb_patterns</i>	Number of hidden patterns	3	5	25
<i>pattern_sup</i>	Support of each hidden pattern	100	100	100
<i>out_factor</i>	Proba. of a pattern labeled –	0.1	0.1	0.1
<i>noise_rate</i>	Proba. of a object to be noisy	0.1	0.1	0.1

positively labeled with a probability of $1 - noise_rate$ to be covered by the description d_i , and $noise_rate$ for not being covered. We also add $pattern_sup \times out_factor$ negative examples for the pattern d_i : It will allow patterns with different WRAcc. Finally, we add random objects until we reach a maximum number of transactions nb_obj .

6.2 Experimental framework

We perform a large pool of experiments to assess this new exploration method for pattern mining. For that, we have designed an experimental framework that enables to test the different combinations of factors for all the strategies we introduced in previous sections. Each experiment are run on the benchmark datasets. An experiment consists in varying a unique strategy parameter while the others are fixed. Since MCTS4DM uses random choices, each experiment is run five times and only the mean of the results is discussed.

Default parameters For each benchmark dataset, we provide a set of default parameters (Table 5). Indeed, due to the specific characteristics of each dataset, a common set of default parameters is unsuitable. Nevertheless, all datasets share a subset of common parameters:

- The maximum size of the result set is set to $maxOutput = 50$.
- The maximum redundancy threshold is set to $\Theta = 0.5$.
- The maximum description length is set to $maxLength = 5$. This is a widely used constraint in SD that enables to restrict the length of the description, i.e., it limits the number of *effective* restrictions in the description.
- The quality measure used is $\varphi = WRAcc$ for the first label only.
- The SP-MCTS is used as the default UCB.
- The permutation unification (PU) strategy is used by default.
- The refinement operator for EXPAND is set to *tuned-min-gen-expand*.
- The *direct-freq-roll-out* strategy is used for the ROLL-OUT
- The reward aggregation policy is set to *max-reward*.
- The memory policy is set to *top-1-memory*.

Table 5 The default parameters for each dataset

Dataset	minSupp	# iterations	Path policy
<i>Bibtex</i>	50	50k	<i>direct-freq-roll-out</i>
<i>BreastCancer</i>	10	50k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>Cal500</i>	10	100k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>Emotions</i>	10	100k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>Ionosphere</i>	10	50k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>Iris</i>	10	50k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>Mushroom</i>	30	50k	<i>direct-freq-roll-out</i>
<i>Nursery</i>	50	100k	<i>direct-freq-roll-out</i>
<i>Olfaction</i>	10	100k	<i>large-freq-roll-out (jumpLength = 30)</i>
<i>TicTacToe</i>	10	100k	<i>direct-freq-roll-out</i>
<i>Yeast</i>	20	100k	<i>large-freq-roll-out (jumpLength = 30)</i>

- The update policy is set to *max-udpate*.

List of experiments Evaluating MCTS4DM is performed with six different batches of experiments:

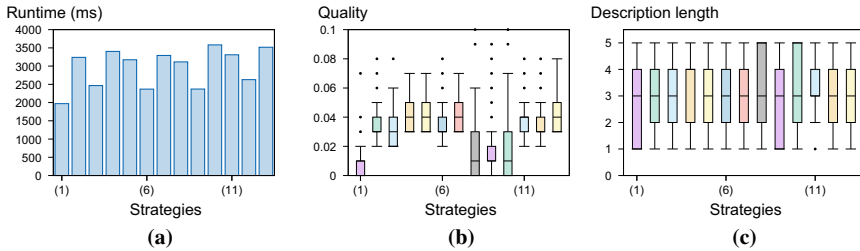
- Section 6.3 is about the choice of the UCB.
- Section 6.4 deals with the several strategies for the EXPAND method.
- Section 6.5 presents the leverage of all the possibilities for the ROLLOUT.
- Section 6.6 shows out the impact of the MEMORY strategy.
- Section 6.7 compares the behaviors of all the strategies for the UPDATE.
- Section 6.8 performs the experiments by varying the computational budget.
- Section 6.9 studies if MCTS4DM is able to retrieved a diverse pattern set.

For simplicity and convenience, for each experiment we display the same batch of figures. For each dataset we show (i) the boxplots of the quality measure φ of the subgroups in the result set, (ii) the histograms of the runtime and (iii) the boxplots of the description length of the subgroups in the result set depending on the strategies that are used. In this way, the impacts of the strategies are easy to understand.

We do not evaluate memory consumption in this section, as it increases linearly with the number of iterations (to which should be added the number of patterns kept by the memory policy).

6.3 The SELECT method

The choice of the UCB is decisive, because it is the base of the exploration/exploitation trade off. Indeed, the UCB chooses which part of the search tree will be expanded and explored. We presented four existing UCBs and an adaptation with a *normalized exploration rate* to take into account an enumeration based on a lexic order (LO). As such, we need to consider also the *expand* methods (standard, *lectic order* LO and *permutation unification* PU) at the same time.



Select Policies: (1) DFS-UCT with LO

(2) UCB1 (3) UCB1 with LO (4) UCB1 with PU

(5) SP-MCTS (6) SP-MCTS with LO (7) SP-MCTS with PU

(8) UCB1-Tuned (9) UCB1-Tuned with LO (10) UCB1-Tuned with PU

(11) UCT (12) UCT with LO (13) UCT with PU

Fig. 6 Impact of the SELECT strategy. **a** Runtime: *BreastCancer*. **b** Avg. quality: *Emotions*. **c** Descr length: *Mushroom*

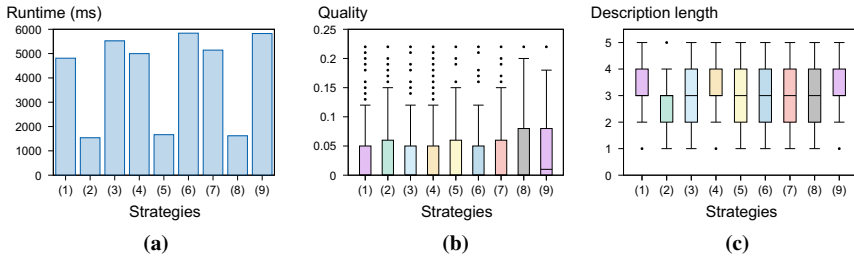
Figure 6 presents the results. Comparing the runtime for all the strategies leads to conclude that there is no difference in the computation of the several UCBs (see Fig. 6a). Indeed, the impact of the UCBs lies in its computation, and there is no UCB that is more time-consuming than others. The difference we can notice, is that when LO is used, the runtime is lower. This result is expected because with LO, the search space is less large since each subgroup is unique in the search space (this is not due to the chosen UCB). PU has also a smaller search space, but it requires call to updates pointers towards subgroups with the same extent, and requires thus more time.

Figure 6b depicts the boxplots of the quality measure of the result set when varying the UCB. The results suggest that the *UCB1-Tuned* and *DFS-UCT* lead to weaker quality result for several datasets: On the *Cal550*, *Emotions* and *Yeast* datasets, the quality measures of the result set are worse than the results of other UCBs (see, e.g., Fig. 6b). This is due to the fact that the search space of these datasets is larger than the other with many local optima, and the *UCB1-Tuned* is designed to explore less, thus less local optima are found. Besides, the *SP-MCTS* seems to be more suitable for SD problems: The quality is slightly better than other UCBs for the *BreastCancer* and *Emotions* datasets. LO leads to a worse quality in the result set, whereas PU seems to be more efficient.

The use of these different UCBs also do not impact the description length of the subgroups within the result set. For some datasets, the *permutation unification* leads to longer descriptions (see for instance Fig. 6c).

6.4 The EXPAND method

Considering the EXPAND policy, we introduced three different refinement operators, namely *direct-expand*, *gen-expand* and *label-expand*, and we presented two methods, namely LO and PU, to take into account that several nodes in the search tree are exactly



- (1) *direct-expand* (2) *direct-expand* with LO (3) *direct-expand* with PU
 (4) *gen-expand* (5) *gen-expand* with LO (6) *gen-expand* with PU
 (7) *label-expand* (8) *label-expand* with LO (9) *label-expand* with PU

Fig. 7 Impact of the EXPAND strategy. **a** Runtime: *Nursery*. **b** Avg. quality: *Iris*. **c** Descr length: *Mushroom*

the same. The several strategies we experiment with are given in Fig. 7 (bottom). Let us consider the leverage on the runtime of these strategies in Fig. 7a. Once again, using LO implies a decrease of the runtime. Conversely, PU requires more time to run. There is very little difference in the runtime when varying the refinement operator: *direct-expand* is the faster one, and *label-expand* is more time consuming.

Considering the quality of the result set varying the expand strategies, we can assume that the impact differs w.r.t. the dataset (see Fig. 7b). Surprisingly, LO improves the quality of the result set for some datasets (e.g. the *Iris* dataset in Fig. 7b). This contradicts what we observe in the *Emotions* dataset of the previous experiment in Sect. 6.3. Most importantly, the results using *label-expand* are better than other ones in most of the datasets. Actually, this is due that this expand favors pattern with a better accuracy which is part of the WRAcc.

The description length of the extracted subgroups are quite constant when varying the expand strategies (see Fig. 7c). With LO, the description lengths are slightly smaller than with other strategies.

6.5 The ROLLOUT method

For the ROLLOUT step we derived several strategies that combine both the path policy and the reward aggregation policy in Table 6. Clearly, the experiments show that the runs using the direct refinement operator (*naive-roll-out* and *direct-freq-roll-out*) are time consuming (see Fig. 8a). In the *BreastCancer* data, the runtime are twice longer with the direct refinement operator than with the *large-freq-roll-out* path policy. In other datasets (e.g., *Ionosphere* or *Yeast*), the runtime is even more than 3 min (if the run lasts more than 3 min to perform the number of iterations, the run is ignored). Besides, it is clear that the *random-reward* aggregation policy is less time consuming than other strategies. Indeed, with *random-reward*, the measure of only one subgroup within the path is computed, thus it is faster.

Figure 8b is about the quality of the result set. The *naive-roll-out* and *direct-freq-roll-out* path policies lead to the worst results. Besides, the quality of the result set

Table 6 The list of strategies used to experiment with the ROLL-OUT method

Strategy	Path policy	Reward aggregation policy
(1)	<i>naive-roll-out</i> (<i>pathLength</i> = 20)	<i>terminal-reward</i>
(2)	<i>direct-freq-roll-out</i>	<i>max-reward</i>
(3)	<i>direct-freq-roll-out</i>	<i>mean-reward</i>
(4)	<i>direct-freq-roll-out</i>	<i>top-2-mean-reward</i>
(5)	<i>direct-freq-roll-out</i>	<i>top-5-mean-reward</i>
(6)	<i>direct-freq-roll-out</i>	<i>top-10-mean-reward</i>
(7)	<i>direct-freq-roll-out</i>	<i>random-reward</i>
(8)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>max-reward</i>
(9)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>mean-reward</i>
(10)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>top-2-mean-reward</i>
(11)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>top-5-mean-reward</i>
(12)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>top-10-mean-reward</i>
(13)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 10)	<i>random-reward</i>
(14)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>max-reward</i>
(15)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>mean-reward</i>
(16)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>top-2-mean-reward</i>
(17)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>top-5-mean-reward</i>
(18)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>top-10-mean-reward</i>
(19)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 20)	<i>random-reward</i>
(20)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>max-reward</i>
(21)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>mean-reward</i>
(22)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>top-2-mean-reward</i>
(23)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>top-5-mean-reward</i>
(24)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>top-10-mean-reward</i>
(25)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 50)	<i>random-reward</i>
(26)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>max-reward</i>
(27)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>mean-reward</i>
(28)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>top-2-mean-reward</i>
(29)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>top-5-mean-reward</i>
(30)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>top-10-mean-reward</i>
(31)	<i>large-freq-roll-out</i> (<i>jumpLength</i> = 100)	<i>random-reward</i>

decreases with the *random-reward* reward aggregation policy in other datasets (e.g., *Emotions*). Basically, these strategies evaluate only random nodes and thus they are not able to identify the promising parts of the search space. Finally, there are not large differences between other strategies.

As can be seen in Fig. 8c, the description length of the subgroups is not very impacted by the strategies of the ROLL-OUT step. The results of the *random-reward* reward aggregation policy are still different from other strategies: The description length is smaller for the *Mushroom* dataset. Using *large-freq-roll-out* with

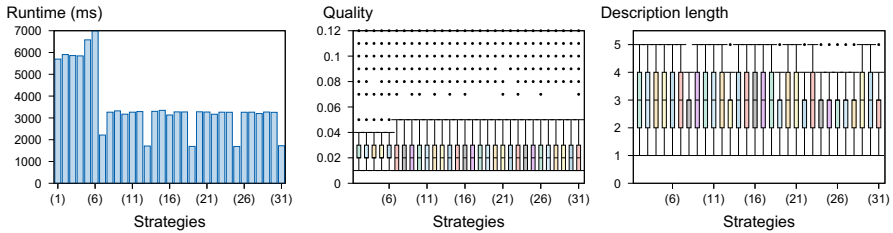


Fig. 8 Impact of the ROLL- OUT strategy. **a** Runtime: *BreastCancer*. **b** Avg. quality: *Mushroom*. **c** Descr length: *Mushroom*

$jumpLength = 100$ leads to smaller descriptions for the *Mushroom* dataset. Finally, the description length is not or almost not influenced by the ROLL- OUT strategies.

6.6 The MEMORY method

We derived six strategies for the MEMORY step given in Fig. 9 (bottom). Obviously, the *all-memory* policy is slower than other strategies because all the nodes within the path of the simulation have to be stored (see Fig. 9a). Conversely, the *no-memory* policy is the fastest strategy. The runtimes of the *top-k-memory* policies is comparable.

Figure 9b shows that the quality of the result set is impacted by the choice of the memory policies. We can observe that the *no-memory* is clearly worse than other strategies. Indeed, in the *Emotion* dataset, the best subgroups are located more deeper in the search space, thus, if the solutions encountered during the simulation are not stored it would be difficult to find them just by considering the subgroups that are expanded in the search tree. Surprisingly, the *all-memory* policy does not lead to better result. In fact the path generated during a simulation contains a lot of redundant subgroups: Storing all these nodes is not required to improve the quality of the result set. Only few subgroups within the path are related to different local optima.

As expected in Fig. 9c, the descriptions of the subgroups obtained with the *no-memory* policy are smaller than those of other strategies. Indeed, with the *no-memory* policy, the result sets contains only subgroups that are expanded in the search tree, in other words, the subgroups obtained with the EXPAND step.

6.7 The UPDATE method

Figure 10 (bottom) presents the different strategies we use to implement the UPDATE step. The goal of this step is to back-propagate the reward obtained by the simulation to the parent nodes. The runtime of these strategies are comparable (see Fig. 10a). However, we notice that the *top-k-mean-update* policy is a little more time consuming. Indeed, we have to maintain a list for each node within the built tree that stores the top-k best rewards obtained so far.

Figure 10b shows the quality of the result set when varying the UPDATE policies. For most of the datasets, since the proportion of local optima is very low within the search space, the *max-update* is more efficient than the *mean-update*. Indeed, using

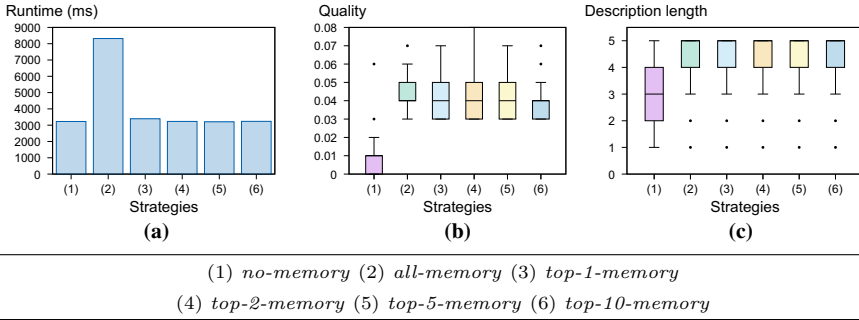


Fig. 9 Impact of the MEMORY strategy. **a** Runtime: *Ionosphere*. **b** Avg. quality: *Emotions*. **c** Descr. length: *BreastCancer*

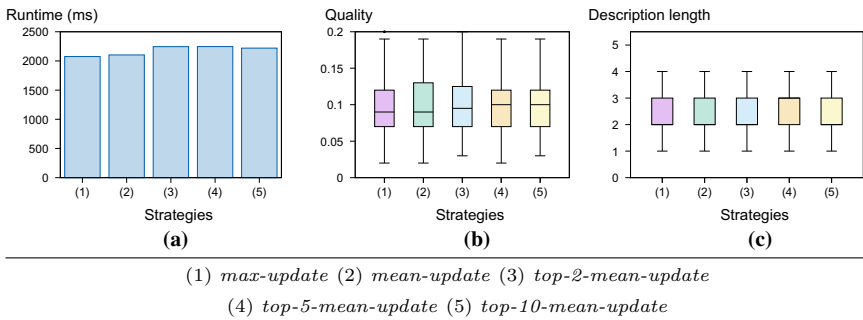


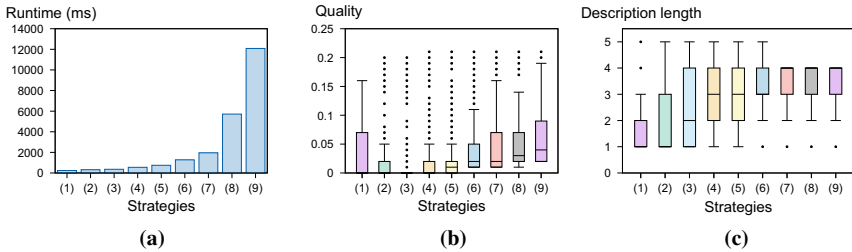
Fig. 10 Impact of the UPDATE strategy. **a** Runtime: *TicTacToe*. **b** Avg. quality: *Ionosphere*. **c** Descr. length: *Iris*

the *max-update* enables to keep in mind that there is an interesting pattern that is reachable from a node. However, Fig. 10b presents the opposite phenomena: The *mean-update* policy leads to a better result. In fact, since there are a lot of local optima in the *Ionosphere* dataset, the *mean-update* can find the areas with lots of interesting solutions. Moreover, using the *top-k-mean-update* leads to the *mean-update* when *k* increases.

The description of the subgroups in the result set is comparable when varying the policies of the UPDATE method (see Fig. 10c). Indeed, the aim of the UPDATE step is just to back-propagate the reward obtained during the simulation to the nodes of the built tree to guide the exploration for the following iterations. This step does not have a large influence on the length of the description of the subgroups.

6.8 The number of iterations

We study the impact of different computational budgets allocated to MCTS4DM, that is, the maximum number of iterations the algorithm can perform. As depicted in Fig. 11a, the runtime is linear with the number of iterations. The x-axis is not linear w.r.t. the number of iterations, please refer to the bottom of Fig. 11 to know the different values of the number of iterations.



(strategy)#iterations: (1)10 (2)50 (3)100 (4)500 (5)1K (6)5K (7)10K (8)50K (9)100K

Fig. 11 Impact of the maximal budget (number of iterations). **a** Runtime: Cal500. **b** Avg. quality: Breast-Cancer. **c** Descr length: Nursery

Moreover, as expected, the more iterations, the better the quality of the result set. Figure 11b shows that a larger computational budget leads to a better quality of the result set, but, obviously, it requires more time. Thus, with this exploration method, the user can have some results anytime. For the *BreastCancer* dataset, the quality decreases from 10 to 100 iterations: This is due to the fact that with 10 iterations there are less subgroups extracted (12 subgroups) than with 100 iterations (40 subgroups), and the mean quality of the result set with 100 iterations contains also subgroups with lower quality measures.

6.9 Evaluating pattern set diversity when a ground truth is known

Artificial datasets are generated according to default parameters given in Table 4. Then, we study separately the impact of each parameter on the ability to retrieve the hidden patterns with our MCTS algorithm. After a few trials, we use the following default MCTS parameters: The *single player UCB* (SP-MCTS) for the select policy; the *label-expand* policy with PU activated; the *direct-freq-roll-out* policy for the simulations, the *max-update* policy as aggregation function of the rewards of a simulation, the *top-10-memory* policy and finally the *max-update* policy for the back-propagation.

The ability to retrieve hidden patterns is measured with a Jaccard coefficient between the support of the hidden patterns and the one discovered by the algorithm:

Definition 17 (*Evaluation measure*) Let \mathcal{H} be the set of hidden patterns, and \mathcal{F} the set of patterns found by an MCTS mining algorithm, the quality, of the found collection is given by:

$$qual(\mathcal{H}, \mathcal{F}) = avg_{\forall h \in \mathcal{H}}(max_{\forall f \in \mathcal{F}}(Jaccard(ext(h), ext(f))))$$

that is, the average of the quality of each hidden pattern, which is the best Jaccard coefficient with a found pattern. We thus measure here the *diversity*. It is a pessimistic measure in the sense that it takes its maximum value 1 iff all patterns have been completely retrieved.

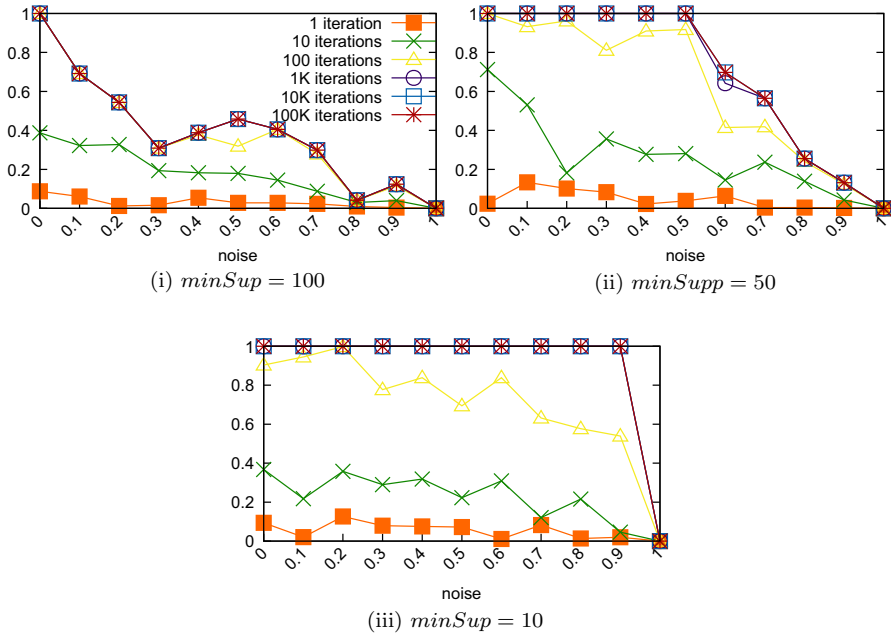


Fig. 12 Ability to retrieve hidden patterns ($qual(\mathcal{H}, \mathcal{F})$ in Y-axis) when introducing noise and mining with different minimum supports $minSup$

It can be noticed that we do not use the Definition 6 for diversity: As a ground truth is available, we opt for a measure that quantifies its recovering.

Varying the noise parameter We start with the set of parameters \mathcal{P}_{small} . The results are given in Fig. 12 with different minimal support values (used during the expand step and the simulations). Recall that a hidden pattern is random set of symbols $attribute = value$ when dealing with nominal attributes, repeated in $pattern_sup$ object descriptions: The noise makes that each generated object description may not support the pattern. Thus, the noise directly reduces the support of a hidden pattern: increasing the noise requires to decrease the minimal support of the algorithm. This is clearly observable on the different figures. When the minimum support is set to the same value as the support of the hidden patterns ($minSup = 100$), the noise has a strong impact and it is difficult to retrieve the hidden patterns, even when the whole tree (of frequent patterns) has been expanded. Reducing the minimal support to 1 makes the search very resistant to noise. Note that when two lines exactly overlaps, it means that the search space of frequent patterns was fully explored: MCTS performed an exhaustive search.

Varying the out factor parameter Each pattern is inserted in $pattern_sup$ transactions (or less when there is noise) as positive examples (class label +). We also add $pattern_sup \times out_factor$ negative examples (class label -). When $out_factor = 1$, each pattern appears as much in positive and negative examples. This allows to hide patterns with different quality measure, and especially different

WRAcc measures. The Fig. 13 [row (1)] shows that this parameter has no impact: patterns of small quality are retrieved easily in a small number of iterations. The UCB hence drives the search towards promising parts that have the best rewards.

Varying the number of hidden patterns We claim that the UCB will guide the search towards interesting parts (exploitation) but also unvisited parts (exploration) of the search space. It follows that all hidden patterns should be retrieved and well retrieved. We thus vary the number of random patterns between 1 and 20 and observe that they are all retrieved [Fig. 13 (row (2))]. When increasing the number of hidden more patterns, retrieving all of them requires more iterations in the general case.

Varying the support size of the hidden patterns Patterns with a high support (relative to the total number of objects) should be easier to be retrieved as a simulation has more chance to discover them, even partially. We observe that patterns with small support can still be retrieved but it requires more iterations to retrieve them in larger datasets [Fig. 13 (row (3))].

Varying the number of objects The number of objects directly influences the computation of the support of each node: Each node stores a *projected database* that lists which objects belong to the current pattern. The memory required for our MCTS implementation follows a linear complexity w.r.t. the number of iterations. This complexity can be higher depending on the chosen memory policy (e.g. in these experiments, the top-10 memory policy was chosen). The time needed to compute the support of a pattern is higher for larger dataset, but it does not change the number of iterations required to find a good result. This is reported in [Fig. 13 (row (4))]. Run times will be discussed later.

Varying the number of attributes and the size of attributes domains These two parameters directly determine the branching factor of the exploration tree. It takes thus more iterations to fully expand a node and to discover all local optima. Here again, all patterns are well discovered but larger datasets require more iterations [Fig. 13 (row (5) and (6))].

7 Comparisons with existing algorithms

We compare MCTS4DM to other SD approaches (exhaustive search, beam search, genetic algorithms and sampling) in terms of computational time, diversity and redundancy of the pattern set, and memory usage. In addition to the benchmark data we used in the previous section, we generate 5 new artificial datasets for which parameters are given in Table 7. In this empirical study, we consider a timeout of 5 min that is enough to capture the behavior of the algorithms that are not based on a computational budget, such as SD-MAP or beam search approaches. Indeed, MCTS4DM and sampling methods use a computational budget.

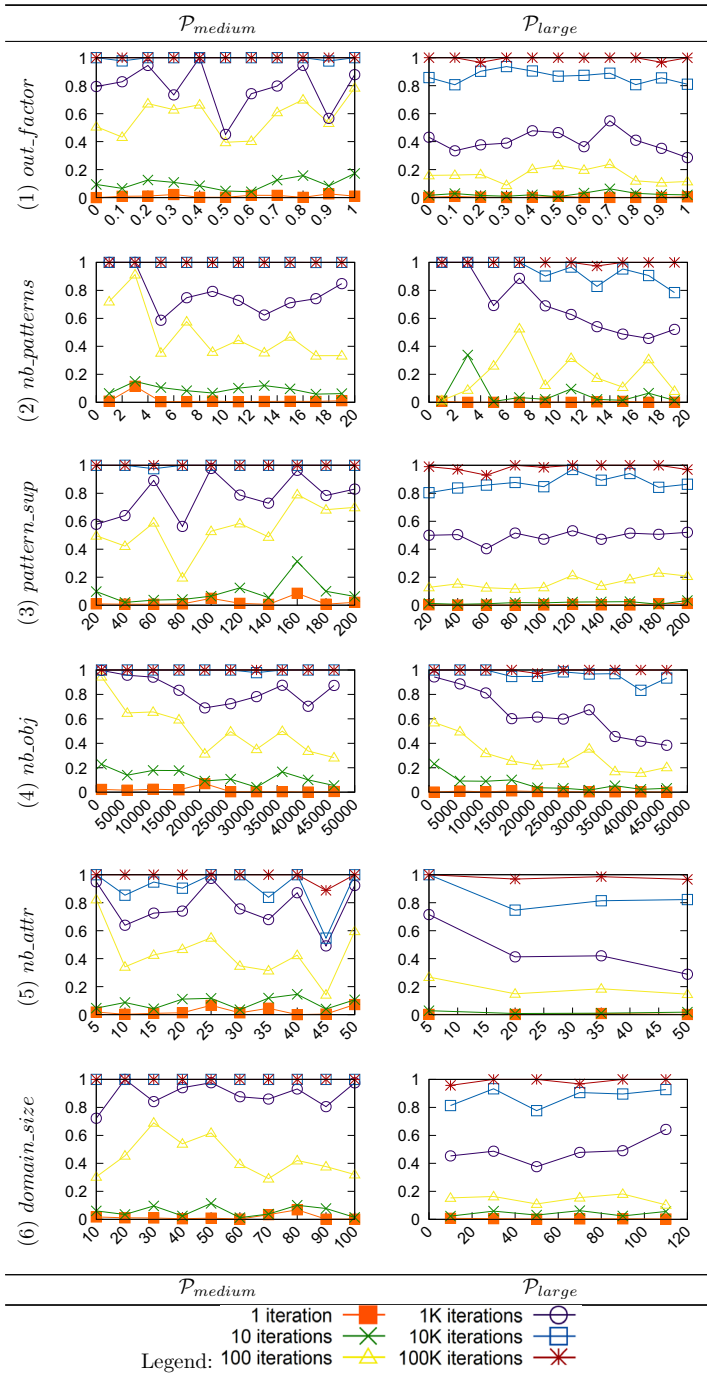


Fig. 13 Evaluation of the ability to retrieve hidden patterns in artificial data generated according to different parameters (average of 5 runs for each point). $qual(\mathcal{H}, \mathcal{F})$ in Y-axis, parameters given in the first columns as X-axis

Table 7 Parameters of the artificial data generator. The format of name of the data is given by their parameters by $[nb_obj]_{[nb_attr]}_{[domain_size]}$

Name	5000_10_200	5000_50_50	5000_50_200	20000_10_200	20000_50_200
nb_obj	5000	5000	5000	20,000	20,000
nb_attr	10	50	50	10	50
$domain_size$	200	50	200	200	200
$nb_patterns$	5	5	5	5	5
$pattern_sup$	200	200	200	200	200
out_factor	0.05	0.05	0.05	0.05	0.05
$noise_rate$	0.05	0.05	0.05	0.05	0.05

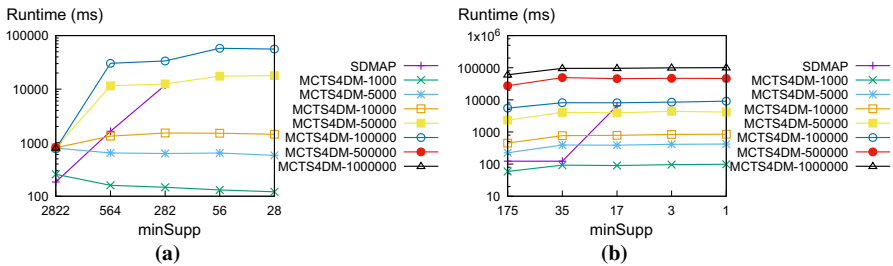


Fig. 14 Runtime of SD- MAP and MCTS4DM when varying $minSupp$. **a** *Mushroom*. **b** *Ionosphere*

7.1 SD-MAP

SD- MAP*, an improvement of SD- MAP, is considered as the most efficient exhaustive method for subgroup discovery (Atzmüller and Puppe 2006; Atzmüller and Lemmerich 2009). It employs the FP-Growth principle to enumerate the search space (Han et al. 2004). It operates a greedy discretization as a pre-processing step to handle numerical data. It can consider several quality measures to evaluate the interestingness of a subgroup ($WRAcc$, $F1$ score, etc.). The source code is available at <http://www.vikamine.org>.

Runtime SD- MAP* is very efficient when dealing with dataset of reasonable search space size. We empirically study the scalability of this algorithm compared to those of MCTS4DM for several numbers of iterations. Figure 14a displays the runtime of SD- MAP* on the *Mushroom* dataset when varying the minimum support threshold. Clearly, for high minimum support thresholds, SD- MAP* is able to provide the results quickly. However, the runtime is exponential w.r.t. this threshold, and thus this algorithm cannot be applied to extract small subgroups. Conversely, MCTS4DM is tractable for very low minimum support thresholds: Many iterations can be performed. Figure 14b displays the runtimes for the *Ionosphere* data and once again MCTS4DM is able to perform lots of iteration in a linear time w.r.t. the minimum support threshold whereas SD- MAP* fails.

Table 8 Diversity in the result set for artificial data

Data	minSupp			SD- MAP			MCTS4DM			BEAMSEARCH			MISERE			SSDP							
	1K	5K	10K	50K	100K	500K	1000K	10	100	500	1K	5K	10K	50K	100K	500K	1000K	100	500	1K	5K	10k	
5000_10_200	1	1	1	1	1	1	1	1	0.38	1	1	1	1	1	1	1	1	0.72	0.72	0.72	0.72	0.72	0.72
5000_50_50	-	0.82	1	1	1	-	-	0.99	1	-	0.73	0.94	0.93	0.99	0.99	0.99	1	0.48	0.72	0.72	0.72	0.72	0.72
5000_50_200	-	1	1	1	1	1	1	1	1	-	0.7	0.89	0.97	0.98	0.98	0.99	1	0.68	0.72	0.72	0.72	0.72	0.72
20000_10_200	1	1	1	1	1	1	1	1	1	-	0.77	0.89	0.96	0.97	1	1	-	0.73	0.73	0.73	0.72	0.72	-
20000_50_200	-	0.69	1	1	1	-	-	-	-	-	0.6	0.86	0.87	0.95	0.98	-	-	0.61	0.72	0.72	0.72	0.72	-

The value is the $qual(\mathcal{H}, \mathcal{F})$ where \mathcal{H} is the set of hidden patterns in the artificial data and \mathcal{F} is the set of found patterns by the algorithm. The character '-' means that the algorithm exceeds the time limit of 5 min

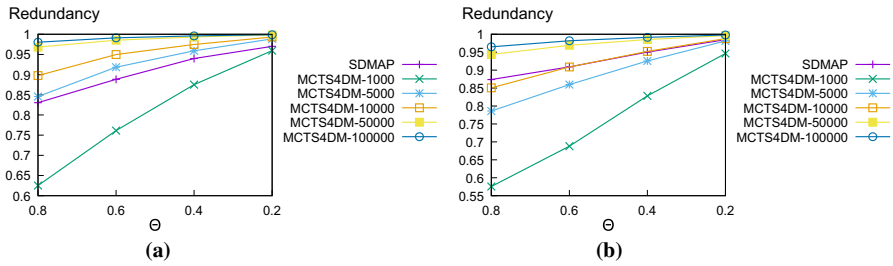


Fig. 15 The redundancy in the result set for the *Mushroom* data varying Θ . **a** $\text{minSupp} = 564$. **b** $\text{minSupp} = 282$

Redundancy and diversity in the result set SD-MAP* is an exhaustive search, thus the diversity of the result set is either perfect if the run can finish or null: Table 8 gives the diversity using the formula of Definition 17 on artificial data since the ground truth is known. However when dealing with numerical attributes, SD-MAP* does not ensure a perfect diversity anymore. Indeed, since it handles numerical attributes by performing a discretization in a pre-processing step, there is no guarantee to extract the best patterns. For example, in the *BreastCancer* dataset, the quality measure of the best subgroup extracted by SD-MAP* is 0.18 whereas MCTS4DM has found a subgroup whose quality measure is 0.21 with 50,000 iterations in only 0.213 ms. Figure 15a, b show the redundancy of the result set (computed with the formula in Definition 5) extracted on the *Mushroom* dataset respectively with $\text{minSupp} = 264$ and $\text{minSupp} = 282$. Obviously, the lower the maximum similarity threshold Θ , the more redundant the result set. Compared to SD-MAP*, MCTS4DM produces few redundancy when performing few iterations, but few iterations are not enough to provide good results: The more iterations, the more redundancy. Surprisingly, the result set of MCTS4DM can be more redundant than those of SD-MAP* that represents our baseline. Indeed, the set of redundant patterns for the main local optima is larger than for other small local optima, i.e., there are many more patterns that are similar with the main local optima than with small local optima. Since MCTS4DM generally finds at first the main local optima, the redundancy measure is higher than those of SD-MAP* because there are, in proportion, more redundant subgroups in the result set than local optima. When minSupp decreases, SD-MAP* becomes more redundant compared to some MCTS4DM runs.

Memory usage Figure 16a displays the memory usage of our algorithm MCTS4DM on the *Mushroom* data with different numbers of iterations. As expected, the more iterations, the higher the memory usage. It grows linearly with the number of iterations (the creation of the storage structures avoids to see the linear growth of the memory during the first iterations). Figure 16b displays the memory usage when varying the minimum support threshold in the *Mushroom* dataset for all the considered algorithms. Here, we only discuss the case of SD-MAP* compared to MCTS4DM. Although SD-MAP* is an exhaustive search, its memory usage is similar to (but slightly lower than) those of MCTS4DM with 100k iterations. It confirms that the implementation of SD-MAP* is efficient.

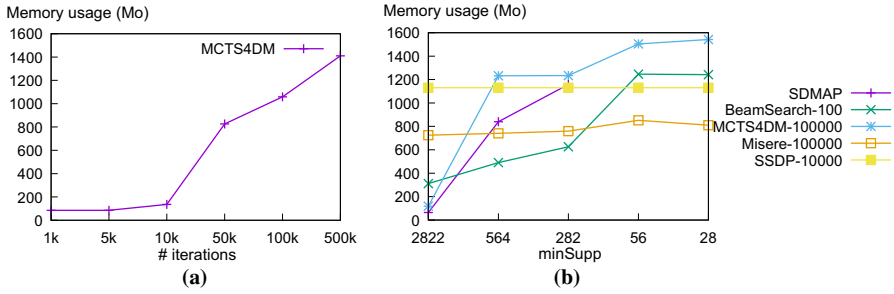


Fig. 16 The memory usage in the *Mushroom* dataset. **a** MCTS4DM. **b** All algorithms

7.2 Beam search

The beam search strategy is the most popular heuristic method in subgroup discovery. Cortana is a tool that enables to run SD tasks with beam search approaches and its source code is available at <http://datamining.liacs.nl/cortana.html>. Beam search, originally introduced in Lowerre (1976), is a greedy method that partially explores the search space with several hill climbings run in parallel. It proceeds in a level-wise approach considering at each level the best subgroups to extend at the next level. The number of subgroups that are kept to be extended at the next level is called the beam width.

Runtime By definition, beam search can only find, yet very quickly, local optima reachable from the most general pattern with a hill climbing. Figure 17 shows the runtimes with different beam widths. Obviously, the larger the beam width, the longer the runtimes. However, even with a beam width of 500, the runtime is lower than those of MCTS4DM with 100k iterations. This is due to the greedy nature of beam search that expands subgroups only if the quality measure increases. However, the local optima that are located deeper in the search space are often missed since the quality measure is not monotone. For large data, such as *Bibtex*, the beam search is not tractable since it is required to expand all the first level of the search tree to build up the beam. Thus, in our settings, the timeout of 5 min is reached with beam search whereas MCTS4DM can proceed to 100k iterations in 4 min.

Redundancy and diversity in the result set Due to the greedy approach of the beam search, the redundancy in the result set is the main problem. Figure 18a compares the redundancy in the *TicTacToe* data obtained with several beam searches and with MCTS4DM. Clearly, the beam search leads to a more redundant result set than MCTS4DM. This remark holds for all data we experimented with. For example, in the *Olfaction* dataset, there is a high difference in the redundancy in the result set obtained with a beam search as well (Fig. 18b). Besides this high redundancy in the result set with a beam search, the diversity is not as good as with MCTS4DM. Even if in Table 8, the beam search extracts the local optima for some datasets, it may require large beam widths that are time consuming. Figure 19 illustrates the diversity for the *BreastCancer* data

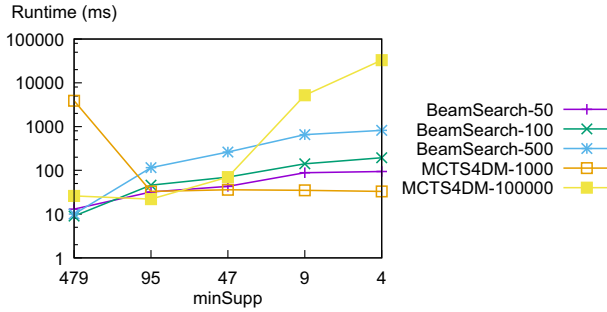


Fig. 17 Runtime of the beam search exploration and MCTS4DM when varying *minSupp* in the *tictactoe* dataset

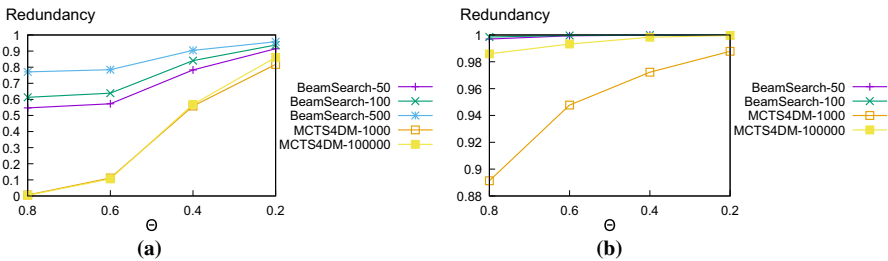


Fig. 18 The redundancy in the result set for the *TicTacToe* and *Olfaction* data for the beam search strategy. **a** *TicTacToe* (*minSupp* = 47). **b** *Olfaction* (*minSupp* = 84)

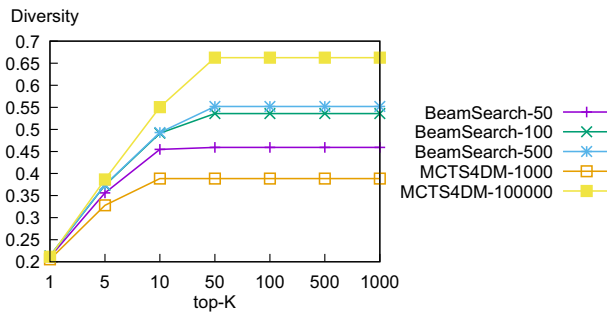


Fig. 19 The diversity of the result set in *BreastCancer* data when $\Theta = 0.2$

with $\Theta = 0.2$. With 100k iterations, MCTS4DM leads to a much more diverse result set than a beam search.

Memory consumption The size of the set of patterns extracted with a beam search is generally lower than the size of the set of patterns obtained with tens of thousands iterations with MCTS4DM. Thus, the memory usage is lower for a beam search. Figure 16 displays the memory usage of a beam search with a beam width set to 100 in the *Mushroom* data. We observe that the memory usage increases similarly to (but is lower than) those of MCTS4DM when varying the minimum support thresholds.

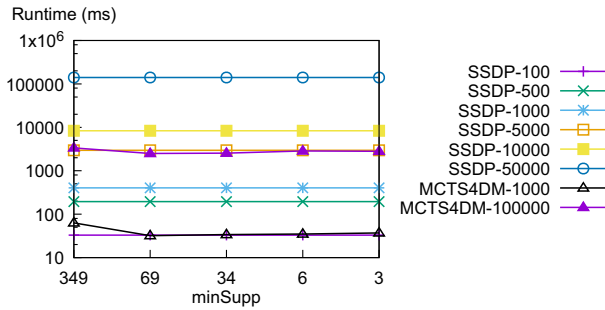


Fig. 20 Runtime of SSDP and MCTS4DM when varying $minSupp$ in *BreastCancer*

7.3 Evolutionary algorithms

The evolutionary approaches aim at solving problems imitating the process of natural evolution. Genetic algorithms are a branch of the evolutionary approaches that use a fitness function to select which individuals to keep at the next generated population (Holland 1975). In this empirical study, we evaluate the efficiency of MCTS4DM from Lucas et al. (2017) against the evolutionary algorithm SSDP.

Runtime SSDP is free from the minimum support constraint: It explores the whole search space without pruning w.r.t. the support of the patterns. Therefore, the runtimes of SSDP are the same for all minimum support thresholds (Fig. 20). However, when varying the population size, it comes with large changes in the runtimes. The runtimes of SSDP are quite similar to those of MCTS4DM when varying the number of iterations. However, in general SSDP is not scalable when considering a large population size.

Redundancy and diversity in the result set On one hand, SSDP seems to provide less redundant pattern sets, due to the mutation and cross-over operations of this evolutionary algorithm. Figure 22a deals with the *Iris* data with $minSupp = 7$: The redundancy of SSDP is generally better than those of our algorithm MCTS4DM. This is the same conclusion in Fig. 21b for *Mushroom* with $minSupp = 56$. On the other hand, the diversity in the result set of SSDP is lower than those of MCTS4DM. In Table 8, SSDP fails to extract all hidden patterns in our artificial data. Figure 22a, b display the same result for benchmark datasets. Clearly, MCTS4DM is able to extract much more interesting subgroups than SSDP. Thus, even if the result set of MCTS4DM can be redundant, it provides a more diverse set of patterns compared to the result set extracted by SSDP. This is due to the population size that is not enough large to provide a high diversity (but SSDP is not tractable for large population sizes).

Memory consumption The memory usage of SSDP depends on the size of the population. In *mushroom*, when considering a population of size 1000, the memory usage is higher than those of MCTS4DM with 100k iterations for high minimum support thresholds but it is lower for low minimum support thresholds (see Fig. 16). Indeed, since SSDP does not use any minimum support threshold, its memory usage is independent w.r.t. $minSupp$.

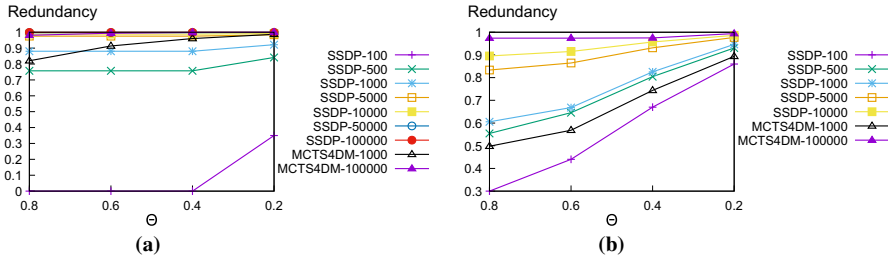


Fig. 21 The redundancy in the result set for the iris and mushroom data. **a** *Iris* ($minSupp = 7$). **b** *Mushroom* ($minSupp = 56$)

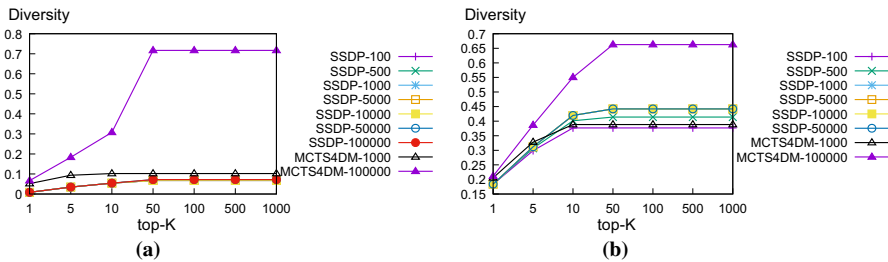


Fig. 22 The diversity of the result set when $\Theta = 0.2$. **a** *Emotions* ($minSupp = 5$). **b** *BreastCancer* ($minSupp = 34$)

7.4 Sampling approach

Sampling methods are useful to provide interactive applications. Indeed, they enable a result anytime. We experiment with the sampling algorithm MISERE (Gay and Boullé 2012; Egho et al. 2015, 2017). Its principle consists in drawing uniformly an object from the data, and then uniformly pick one of its possible generalizations. Each sample is independent and thus a pattern can be drawn several times. We chose MISERE as it can consider any pattern quality measure [in contrast to other sampling approaches such as Moens and Boley (2014) and Boley et al. (2011)], and it performs very well.

Runtime Since this strategy consists in randomly drawing patterns, the runtime is linear with the number of draws. Varying the minimum support thresholds does not really impact the runtime (Fig. 23). An iteration with MCTS4DM is almost only twice much longer than a draw with MISERE. This is explained by the fact that MISERE only draws one pattern at once without additional memory (i.e., the Monte Carlo tree). Conversely, in one iteration, MCTS additionally performs SELECT, EXPAND, MEMORY and UPDATE steps.

Redundancy and diversity in the result set Since MISERE proceeds in independent draws of patterns without exploiting the result of the previous draws of patterns, it leads to a result set that contains little redundancy. Indeed, it can draw an interesting pattern that is close to its local optimum, but it would not try to find this optimum at the next draws. Figure 24 illustrates this: The result set is much less redundant than those of MCTS4DM. For example, considering a result set containing 1000 draws of patterns

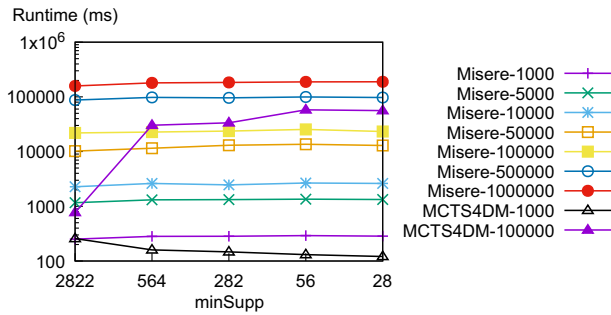


Fig. 23 Runtime of MISERE and MCTS4DM when varying *minSupp* on the *Mushroom* dataset

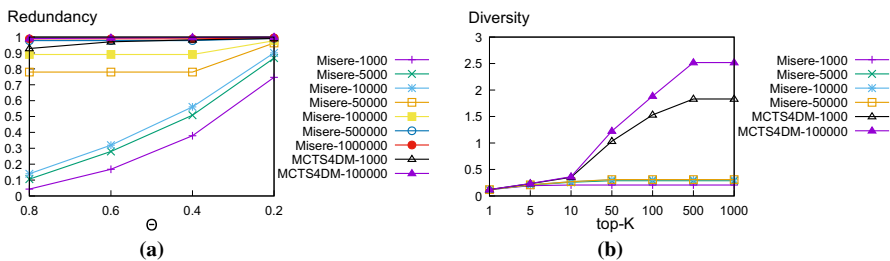


Fig. 24 The redundancy and the diversity in the result set for the *Cal500* and *Bibtex* data. **a** Redundancy in *Cal500*. **b** Diversity in *Bibtex*

and another obtained with 1000 iterations from MCTS4DM. MCTS4DM returns pattern set 10 times more redundant than MISERE. However, since MISERE does not exploit the result of the previous draws, it leads to less diversity. In Table 8, MISERE may require lots of draws to find all local optima. Figure 24b shows for *Bibtex* that the diversity is better with MCTS4DM than with MISERE. Contrary to MCTS4DM, there is no guarantee that MISERE will explore the whole search space, even given a large computational budget. Nevertheless, in Table 8, we can notice that, in practice, MISERE can extract all patterns hidden in artificial data, but it might require a lot of draws to find them.

Memory consumption As expected, since this sampling method performs independent draws, the memory usage is low. In our settings, only the patterns that are drawn are stored. Figure 16 illustrates the memory usage of MISERE when it has randomly picked 100k patterns. It is constant w.r.t. the minimum support thresholds, and this is the exploration method that requires the less memory for low minimum support thresholds.

7.5 Considering several measures

MCTS4DM can consider any pattern quality measures. Up to now, we experimented with the popular *WRAcc* measure only. We empirically evaluate MCTS4DM with several quality measures that are also used in SD. We consider some of the quality measures available in Cortana: The entropy, the F1 score, the Jaccard coefficient and the accuracy

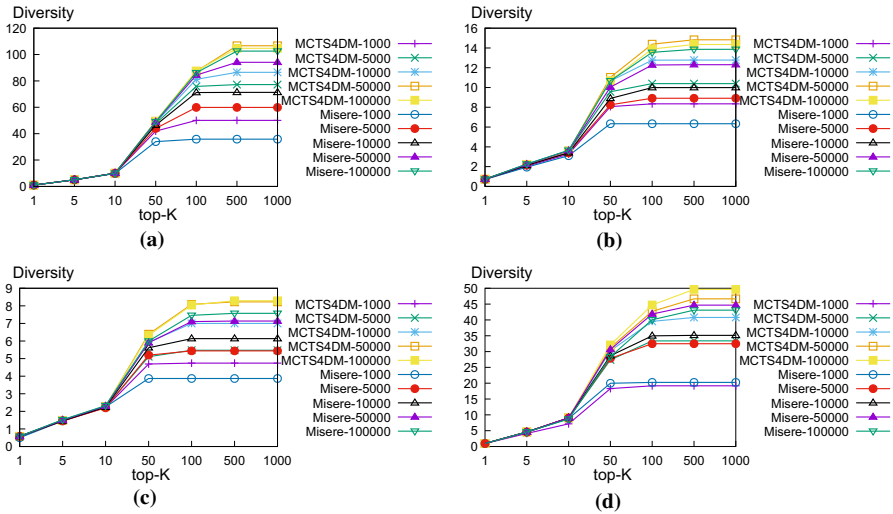


Fig. 25 The diversity of the result set for several quality measures in the *Mushroom* dataset. **a** Entropy. **b** F1. **c** Jaccard coefficient. **d** Accuracy

(or precision). The measures we use are not equivalent since they do not sort the patterns in the same order : Each measure induces a specific profile on the pattern space.

MCTS4DM is not measure-dependent since it does not use any prior knowledge to explore the search space. During the first iterations, MCTS4DM randomly samples the search space, then once it has an estimation—that is usually rather not reliable at the beginning—of the distribution of the quality measure on the pattern space, it biases the exploration to focus on the promising areas (*exploitation*) and the areas that have been rarely visited (*exploration*). The strategies we developed are useful to handle the specific profile induced by a quality measure on the pattern space, e.g., if there are lots of local optima in the search space the exploration strategy should be different than if there are few local optima. For instance, the *mean-update* strategy is the most efficient strategy when we are facing a pattern space with lots of local optima since it enables to exploit the areas that are deemed to be interesting in average. Thus, MCTS4DM can be used with any quality measure. The choice of the strategies only impacts how fast it will find the interesting patterns.

We compare our approach with the sampling method MISERE which is the most efficient opponent based on the previous results (see Fig. 25). We experiment on the *Mushroom* dataset to reach low minimum support thresholds using the four quality measures (the entropy, the F1 score, the Jaccard coefficient and the accuracy). The results suggest that MCTS4DM is able to provide a good diversity regardless the quality measure that is used. MCTS4DM finds a result set with a better diversity for all quality measures. The results on the other datasets are similar but not reported here.

8 Discussion

Based on the empirical study reported in the two previous sections, we now provide a summary of the main results. First, we experimented with several strategies we defined for our algorithm MCTS4DM. Our conclusions are the following:

- **SELECT:** Concerning the choice of the upper confidence bound, it seems more suitable to use the *SP-MCTS* for SD problems, although it has a limited impact. Activating LO leads to worse results, but with PU we are able to get more interesting patterns. This is a quite interesting fact as LO is a widely used technique in pattern mining (enumerate each pattern only once with a lexic order).
- **EXPAND:** We advise to use the *label-gen* strategy that enables to reach more quickly the best patterns, but it can require more computational time.
- **ROLLOUT:** For nominal attributes, the *direct-freq-roll-out* is an efficient strategy. However, when facing numerical attributes, we recommend to employ the *large-freq-roll-out* since it may require a lot of time to reach the maximal frequent patterns.
- **MEMORY:** Using a memory strategy is essential since it enables to store the patterns encountered during the ROLLOUT step. The *top-1-memory* is enough to avoid to miss interesting patterns that are located deeper in the search space.
- **UPDATE:** When there are potentially many local optima in the search space, we recommend to set the *mean-update* strategy for the UPDATE step. Indeed it enables to exploit the areas that are deemed to be interesting in average. However, when there are few local optima among lots of uninteresting patterns, using *mean-update* is not optimal since the mean of the rewards would converge to 0. In place, the *max-update* should be used to ensure that an area containing a local optima is well identified.

Our second batch of experiments compared MCTS4DM with the main existing approaches for SD. For that, we experimented with one of the most efficient exhaustive search in SD, namely SD- MAP*, a beam search, the recent evolutionary algorithm SSDP and a sampling method implemented in the algorithm MISERE. The results suggest that MCTS4DM leads, in general, to a more diverse result set when an exhaustive search is not tractable. The greedy property of the beam search leads to a low diversity in the result set, and the lack of memory in sampling methods avoid to exploit interesting patterns to find the local optima (a pattern may be drawn several times). There is no guarantee that evolutionary algorithms and sampling approaches converge to the optimal pattern set even with an infinite computational budget.

MCTS comes with several advantages but has some limits:

- + It produces a good pattern set anytime and it converges to an exhaustive search if given enough time and memory (a *best-first search*).
- + It is agnostic of the pattern language and the quality measures: It handles numerical patterns without discretization in a pre-processing step and it still provides a high diversity using several quality measures.
- + MCTS4DM is aheuristic: No hypotheses are required to run the algorithm whereas with some sampling methods, a probability distribution (based on the quality measure and the pattern space) has to be given as a parameter.

- MCTS4DM may require a lot of memory. This memory usage becomes more and more important with the increase of the number of iterations.
- Despite the use of UCB, it is now well known that MCTS algorithms explore too much the search space. As MCTS basically requires to expand all the children of a node before exploiting one of them, this problem is even stronger when dealing with very high branching factor (number of direct specializations of a pattern). This problem has been in part tackled by the progressive widening approach that enables to exploit a child of a node before all of the other children of the node have been expanded (Gaudel and Sebag 2010; Browne et al. 2012).

9 Conclusion

Heuristic search of supervised patterns becomes mandatory with large datasets. However, classical heuristics lead to a weak diversity in pattern sets: Only few local optima are found. We advocate for the use of MCTS for pattern mining: An exploration strategy leading to “any-time” pattern mining that can be adapted with different measures and policies. The experiments show that MCTS provides a much better diversity in the result set than existing heuristic approaches. For instance, interesting subgroups are found by means of a reasonable amount of iterations and the quality of the result iteratively improves.

MCTS is a powerful exploration strategy that can be applied to several, if not all, pattern mining problems that need to optimize a quality measure given a subset of objects. For example, Belfodil et al. (2017) have already tuned MCTS4DM for mining convex polygon patterns in numerical data. In general, the main difficulties are to be able to deal with large branching factors, and jointly deal with several quality measures. This opens new research perspectives for mining more complex patterns such as sequences and graphs.

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