3D SHAPE CODING WITH SUPERQUADRICS

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ABSTRACT

We present a geometric coding method for 3D real and virtual objects transmission and visualization. We describe object surfaces with a set of superellipsoids which are structured with a constructive solid geometry (CSG) tree. An unstructured cloud of 3D points lying on the original object surface is considered as initial data. The tree is obtained using a split and merge algorithm. At each splitting step, the set of points is partitioned into two superellipsoids. The parameters of each ellipsoid is computed with a non-linear regression technique (Levenberg-Marquardt). This decomposition is stopped when the estimation error reaches a given threshold (the accepted distortion). The objective of the merging algorithm is to reduce the number of superellipsoids used to describe the complex 3D object without increasing the global distortion. This algorithm allows to merge both connex and non-connex elements of the superellipsoids set obtained from the decomposition step. The whole scheme is applied on synthetic objects.

1. INTRODUCTION

This paper addresses the problem of 3D complex object description using a simple, compact (*low bit rate*), scalable (progressive transmission and visualization) descriptor which leads to a good quality of approximation (the controled distortion).

A descriptor based on superellipsoids [1] is proposed. This surface model needs only a few number of parameters and permits to describe a wide variety of shapes. This surface model has been previously used for anatomic modeling with only one superellipsoid which is globaly deformed with a free form deformation [2]. Some other studies have been based on a set of concurrent superellipsoids for approximating surfaces [3, 4, 5, 6, 7]. In [3], a region growing process has been proposed to reduce the number of superellipsoids.

In the current study, we propose a new segmentation method using a split and merge approach to model a cloud of unstructured 3D data with a set of superellipsoids. Section 2 details our primitive surface descriptor: a deformed superellipsoid. In section 3, we show how 3D data can be approximated using only one deformed superellipsoid. Section 4 gives details on the split and merge algorithm which leads to the final descriptor (a set of superellipsoids structured with a Constructive Solid Geometry (CSG) tree. Finally, the qualitative and quantitative performances of this descriptor are illustrated on synthetic 3D data.

2. THE PRIMITIVE SURFACE MODEL

Superellipsoid model has been introduced in computer graphics by A.H. Barr in 1981 [1]. A superellipsoid is defined as the solution of an implicit equation :

$$f(x,y,z) = \left(\left(\frac{x}{a_1}\right)^{\frac{2}{\epsilon_2}} + \left(\frac{y}{a_2}\right)^{\frac{2}{\epsilon_2}} \right)^{\frac{\epsilon_2}{\epsilon_1}} + \left(\frac{z}{a_3}\right)^{\frac{2}{\epsilon_1}} \quad (1)$$

where

- a_1, a_2, a_3 are scale factors on respectively x, y and z axis
- ϵ_1 and ϵ_2 determine the shape curvatures

and we have

- $f(x,y,z) = 1$ when the point lies on the surface
- $f(x,y,z) < 1$ when the point is inside the superellipsoid
- $f(x,y,z) > 1$ when the point is outside

This is a compact model defined by only 5 parameters that permits to handle a large variety of shapes : ellipsoid ($\epsilon_1 = \epsilon_2 = 1$), parallelepiped ($\epsilon_1 \to 0$ and $\epsilon_2 \to 0$), cylinder ($\epsilon_1 = 1$ and $\epsilon_2 \to 0$)...(Fig. 1).

In order to increase the flexibility of this model, we add two deformation parameters (p_1 and p_2) to represent the 3D objects which present bends and dissimetries (Fig. 2). It acts as if the y axis is defined whit a degree 2 polynom instead of a straight line.

The implicit surface form of the new deformed primitive is defined by :

$$f(x,y,z) = \left(\left(\frac{x}{a_1}\right)^{\frac{2}{\epsilon_2}} + \left(\frac{\mathbf{y + p_1 * x + p_2 * x^2}}{a_2}\right)^{\frac{2}{\epsilon_2}} \right)^{\frac{\epsilon_2}{\epsilon_1}} + \left(\frac{z}{a_3}\right)^{\frac{2}{\epsilon_1}} \quad (2)$$

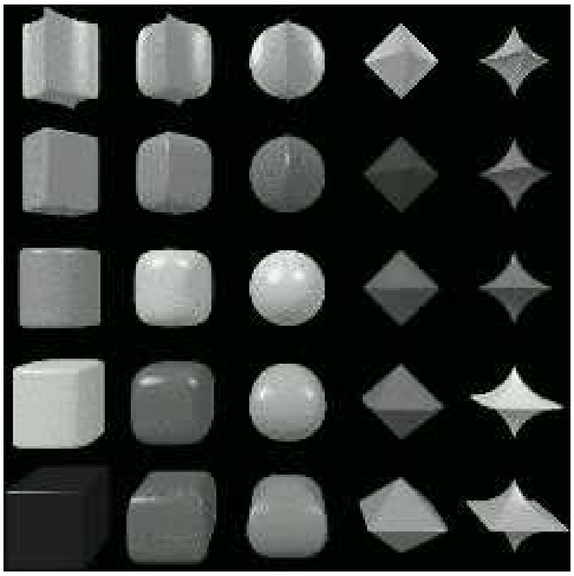

Fig. 1. Examples of superellipsoids.

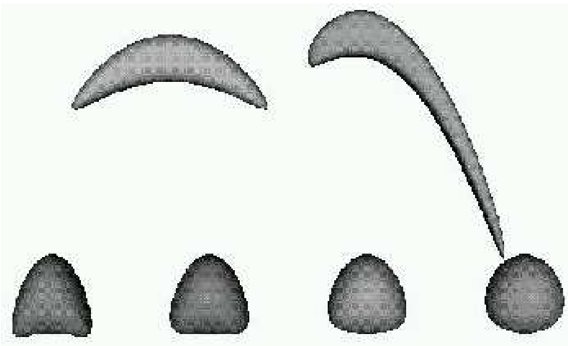

Fig. 2. Examples of deformed superellipsoids.

3. APPROXIMATION OF 3D DATA WITH ONE DEFORMED SUPERELLIPSOID

Given a set of N unstructured 3D data, the challenge is to determine the parameters of our model with a global distortion constraint. For each point, the following distance is minimized :

$$d = f^{\frac{\epsilon_1}{2}} - 1 \quad (3)$$

If the points set is not closed, several superellipsoids may approximate the same set. This difficulty is resolved imposing a constraint which priviligies the small superellipsoids. This is achieved by applying the coefficient $\sqrt{a_1 a_2 a_3}$ to the global distortion normalized by the number of points. This leads to D, the mean distortion per point given by:

$$D = \frac{\sqrt{a_1 a_2 a_3}}{N} \sum \left(f\,(\,x_i, y_i, z_i; a_1, a_2, a_3, \epsilon_1, \epsilon_2, t_x, t_y, t_z, r_x, r_y, r_z)^{\frac{\epsilon_1}{2}} - 1\right)^2 \quad (4)$$

where (t_x,t_y,t_z) and (r_x,r_y,r_z) are translation and orientation parameters respectively. The minimization of D is performed with Levenberg-Marquardt algorithm which consists in a non-linear regression approach. The algorithm converges when the accepted distortion per point T is reached.

4. DESCRIPTION OF 3D DATA WITH A SET OF SUPERELLIPSOIDS

The main motivation of this study is to approximate a set of unstructured 3D points using a structured set of deformed superellipsoids as presented in section 2. The main difficulty consists in partitioning the data in order to obtain a compact set of primitives and to provide high quality local approximation. We propose a split and merge algorithm to solve the problem.

4.1. Split procedure

This recursive procedure consists in the subsequent steps:

1. The set of 3D points is approximated using one deformed superellipsoid (as seen in section 3)
2. If the distortion D is less than the threshold T (the tolerated distortion constraint), the procedure is over. Otherwise, step 3 is processed.
3. The set of 3D points is split into two regions using the plane P orthogonal to the inertia axis of this set (P contains the centroid of the set).
4. Each half-subset is approximated independently using one deformed superellipsoid. For each subset, the procedure is iterated from step 2.

4.2. Merge procedure

This procedure minimizes the number of superellipsoids without increasing the whole distortion. The topological relations between 3D points are taken into account. The following steps are processed:

1. A set of couples (subset, neighbouring subset) is associated to a given subset. The neighbourhood considered here is particulary large: a subset is neighbour of another subset s if it is the direct (connex) neighbour of s (first order neighbourhood) or if it is the neighbour of a direct neighbour of s (second order neighbourhood).

2. For each couple of subsets, the approximation is performed using one deformed superellipsoid. We merge the couple which minimizes the approximation distortion D if this one is less than a threshold T'. We go back to step 3. If all the distortions are greater than T', the process is over.

3. A new set of superellipsoids is obtained. We go back to step 1.

4.3. Final descriptor

The set of superellipsoids issued from the merging algorithm is unstructured. Structuration is obtained using a CSG based tree. The superellipsoids become leaves of this tree. The nodes represent the topological operators between these primitives (union, intersection or difference).

This structuration has at least three advantages. First, it is obvious that the difference operator leads to a more compact descriptor for recovering objects with holes or concavities. Second, the tree structure may allow the simplification of the redundancy in the object description (the elimination of structures which are similar by rotation, translation or scale factor). Third, this structure is well adapted to a progressive transmission and visualization of an object leading to "a coarse to fine" representation. Futhermore, it is also possible to hide a given part of the object by suppressing the corresponding subtree.

5. RESULTS

First experiments are carried out for synthetic 3D objets obtained by combining superquadrics (Fig. 3, 4 ,5). The process takes a few minutes to achieve.

Object	Number of points	Number of superellipsoids after the split procedure	Number of superellipsoids in the final descriptor	Mean distortion
Fig. 3	900	3	2	0.0024
Fig. 4	900	8	3	0.0014
Fig. 5	1500	38	4	0.0260

6. CONCLUSION

A new surface descriptor for unstructured 3D data has been developed. This descriptor does not need any a priori knowledge about the object. A superellipsoid with global deformation is considered as a primitive model for the approximation. The segmentation process is first global to local in order to control the approximation quality using a global distortion constraint. Next, the strategy becomes local to global to insure the descriptor compacity. The set of superellipsoids obtained with the split and merge scheme is also structured (CSG tree) to increase the compacity and the functionality of the model. The efficiency of the description is illustrated with synthetic 3D data.

7. REFERENCES

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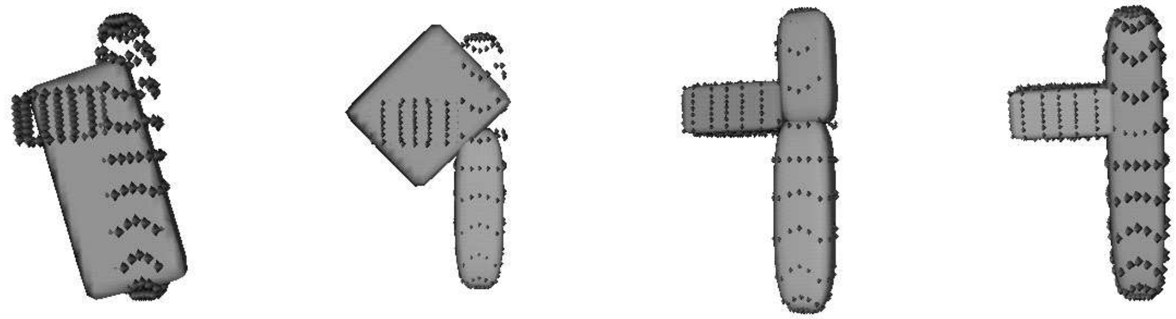
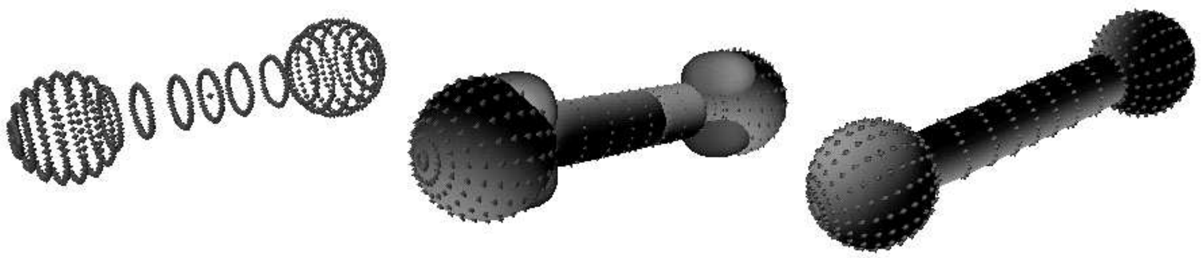


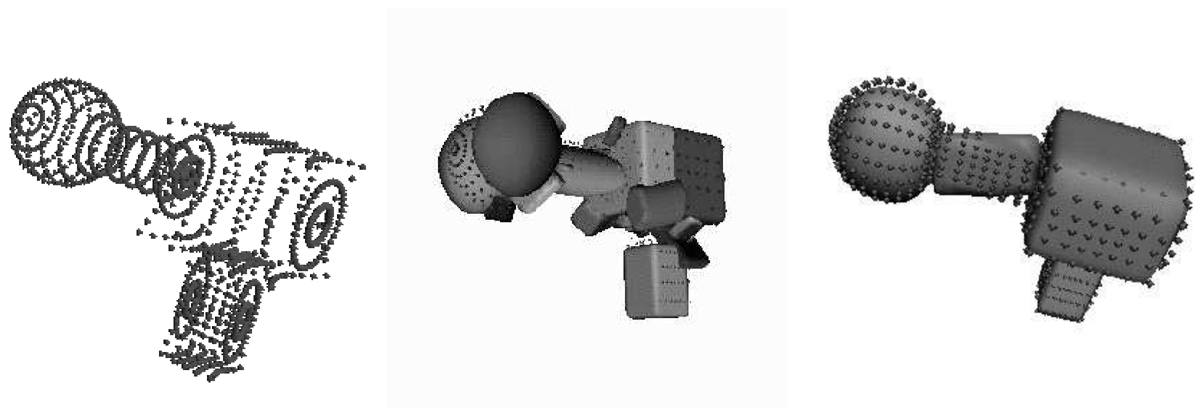
(a) (b) (c) (d)

Fig. 3. (a) Original data and first approximation. (b) First split step. (c) Second split step. All subsets are well approximated. The merge procedure starts. (d) Two superellipsoids are merged. The final descriptor is the union of two superellipsoids.



(a) (b) (c)

Fig. 4. (a) Original 3D data. (b) After the split procedure. (c) The final descriptor (3 superellipsoids).



(a) (b) (c)

Fig. 5. (a) Original 3D data. (b) After the split procedure. (c) The final descriptor (4 superellipsoids).