Periodic B-Spline Surface Skinning Of Anatomic Shapes

Fabrice JAILLET, Behzad SHARIAT and Denis VANDORPE
E-mail : {fjaillet,bshariat,vandorpe}@ligim.univ-lyon1.fr
Address: LIGIM, bat710, Université Lyon I
43 bd du 11 nov. 1918, 69622 VILLEURBANNE Cedex, FRANCE

1 Introduction

In the medical area, great improvements have been made in digital imagery techniques, as computed tomography (CT) or magnetic resonance imagery (MRI). Thus, more and more , the anatomic objects are described as a set of 2D cross-sections.

Here, we are interested in the reconstruction of the organs under investigation. This can be achieved by first extracting from each section a set of closed contours, corresponding to the intersection of the real surface with the sectional plane. Numerous methods have been presented, some of them based on "snakes" [CC90].

Next, the correspondence problem between linked contours on adjacent sections has to be solved [EPO91]. It means that the anatomic shapes have to be segmented into different branches. Then, the external shape has to be fitted with either an interpolation or an approximation surface.

A lot of methods have been proposed for the surface fitting problem. The most widespread approach is the polyhedral model. A set of triangular facets are generated between adjacent contours [FKU77, MSS92]. Since these methods produced a great number of triangles, some re-tiling algorithms have been developed [Tur92]. [JS95] proposes to use the triangulation as a guide to construct a tensor product smooth surface.

Skinning algorithms have been previously proposed for constructing a piecewise rectangular surface, but they present some limitations. In [PK96], the section curves are interpolated, this leads to a great number of control points to ensure the accuracy, while in [PT96] the contours are approximated but they only handle the case of open contours. Both methods are time consuming in order to bring the curves compatible for the skinning.

In this paper, we present a method for smooth closed surface approximation from 2D contours. Both the section curves and the skinned surface are approximated. This allows to reduce dramatically the amount of initial data while preserving the accuracy. Here, we are interested in single contour sections, but the handling of multiple contours is presented in the conclusion. Moreover, the resulting surfaces can easily be manipulated in an interactive system, with the help of the numerous techniques developed for the shape modification of B-spline curves and surfaces.

The organization of the paper is as follows. In section (2), some necessary B-spline formulas are briefly presented. In section (3), the general surface reconstruction method is presented, including a description of the curve fitting and the skinning of the section curves. The section (4) presents some experimental results. Finally the paper is closed by a conclusion section.

2 Closed B-Spline Formulas

A parametric k^{th} degree closed B-spline curve is a piecewise polynomial curve defined by:

$$C(t) = \sum_{i=0}^{m+k-1} P_{i mod(m+1)} N^k_{i mod(m+1)}(t) \qquad (1)$$

for any $t \in \mathbb{R}$, where the $m+1$ distinct P_i are the control points and the N_i^k B-spline basis functions of degree k can be recursively defined as:

$$N_i^0(t) = \left[\begin{array}{ll} 1, & \text{if } t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{array} \right.$$

$$\begin{aligned} N_i^k(t) &= \frac{t - t_i}{t_{i+k} - t_i} N_i^{k-1}(t) \\ &\quad + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} N_{i+1}^{k-1}(t) \end{aligned}$$

with 0/0=0, and defined on the knot vector t_i, which is an increasing sequence of knots.

For a closed B-spline curve, the knot sequence T has the following form:

$$T = \{\underbrace{t_{-k}, \ldots, t_{-1}}_{k}, \underbrace{t_0, \ldots, t_m}_{m+1}, \underbrace{t_{m+1}, \ldots, t_{m+k}}_{k}\}$$

$$\begin{aligned} t_0 &= 0 \text{ and } t_m = 1 \\ t_{-i} &= t_{-i+1} - (t_{m-i+1} - t_{m-i}) \\ t_{m+i} &= t_{m+i-1} + (t_i - t_{i-1}) \\ & \quad (\text{for } i = 1, \ldots, k) \end{aligned} \tag{2}$$

The previous definitions ensure that the curve is closed, $C(0) = C(1)$, and is defined for $t \in \mathbb{R}$ with a C^{k-1} continuity everywhere.

A biparametric closed B-spline surface can be built as a tensor product of closed B-spline curves. The surface defined here is closed in the contour's direction but open in the other one. A surface of degree $k \times l$ can be written as follows:

$$\begin{aligned} & S(u, v) = \\ & \quad \sum_{j=0}^{n} \sum_{i=0}^{m+k-1} Q_{i mod(m+1),j} N^k_{i mod(m+1)}(u) N^l_j(v) \\ & \quad \text{with } (u \in \mathbb{R}) \text{ and } (0 \leq v \leq 1) \end{aligned} \tag{3}$$

where the $(m+1) \times (n+1)$ distinct control points $Q_{i,j}$ define the control net, and the N^k_i, N^l_j are the spline basis functions in the direction u and v respectively. The $N^k_i(u)$ are defined on a knot sequence $U = \{u_{-k}, \ldots, u_{-1}, u_0, \ldots, u_m, u_{m+1}, \ldots, u_{m+k}\}$ and the $N^l_j(v)$ are defined on the knot sequence $V = \{v_0 = \ldots = v_l, v_{l+1}, \ldots, v_n, v_{n+1} = \ldots = v_{n+l+1}\}$

The u parameter curve is a closed curve (in the direction of the contours), while the isoparametric curves in v direction are open (in the longitudinal direction).

3 B-Spline Surface Reconstruction

In this section, we describe the proposed method for the surface reconstruction from a set of planar cross-sections, each of them containing only one contour. First, we fit a closed B-spline curve to each polygonal contour. Then we perform the surface skinning of these section curves. This necessitates that all the section curves are defined on a common knot vector.

3.1 Closed curve fitting

The purpose of this section is the construction of a curve from the points of a planar contour. This problem has been widely studied in previous research works [Leo91, PT95]. We want to create a B-spline closed curve of degree k which approximates $p+1$ given contour points R_i. The number of control points, $m+1$, should be specified, as well as an appropriate knot vector $T = \{t_{-k}, \ldots, t_{-1}, t_0, \ldots, t_m, t_{m+1}, \ldots, t_{m+k}\}$. The degree and the number of control points could generally be refined using an iterative least-square approximation.

We can rewrite the equation (1) for each point R_i of the contour in matrix form: $[R] = [N][P]$, where $[R]$ is a vector of dimension $(p+1)$ representing the known contour points, $[N]$ is the B-spline basis coefficient matrix of dimension $(p+1) \times (m+1)$, and $[P]$ is the vector of dimension $(m+1)$ of the unknown control points of the B-spline curve.

Generally, the matrix $[N]$ is not square $(m < p)$ and the linear equations system can only be solved with a least-square approximation method [RF89]. The resolution of $[P] = [[N]^t[N]]^{-1}[N]^t[R]$ gives the control points which minimize the error on the curve.

For this, we should assign a parameter value $\overline{t}_i$ to each contour point R_i, which should take into account its position on the curve. The determination of this parameterization is very important in curve reconstruction [MK95], since a bad parameter value or knot vector can produce unwanted oscillations. The uniform parameterization is not recommended when the data are unevenly spaced, thus we prefer the chord-length method intensively employed in CAD.

The following equations [PT95] guarantee that every knot span $[t_i, t_{i+1}]$ contains at least one $\overline{t}_j$. This condition ensures that the system can be solved by Gaussian elimination without pivoting. The internal knots t_i for $i = 0, \ldots, m$ are defined as follows:
let $d = (p+1)/(m+1)$, then

$$\begin{cases} t_0 = 0 \,;\, t_m = 1 \\ t_i = (1-\alpha).\overline{t}_j + \alpha.\overline{t}_{j+1} \quad (i = 1, \ldots, m-1) \end{cases}$$

where $j = int(id)$ and $\alpha = id - j$.

3.2 Curve compatibility

To achieve the final surface reconstruction, we should fit a closed B-spline curve to each set of contour points. Then, due to its tensor product formulation, the surface skinning requires that all the $q+1$ curves have the same degree and defined on the same knot vector.

The first condition can be achieved easily by elevating the degree of each curve to the maximum of the degrees.

In return, it is not trivial to define a common knot sequence for all the curves, because the knot placement will affect the whole surface along the longitudinal direction. In [PT96], the authors have proposed an algorithm to merge the different knot vectors into a common one. But the resulting vector can contain thousands of points, necessitating a further treatment to eliminate a lot of knots while respecting a given tolerance. Since making the knot vectors compatible is very time consuming, if we assume that the contours are not too different in shape, we can simply average the internal knots:

$$u_j = \frac{1}{q+1}\sum_{i=0}^{q} u_j^i \qquad (j = 0, \ldots, m)$$

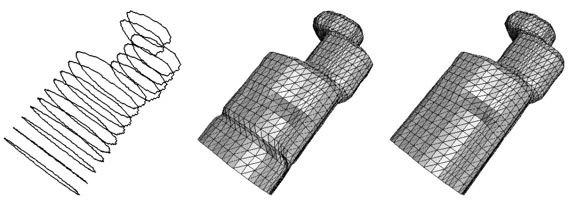
where the u_j^i are the j^{th} internal knot of the i^{th} curve. It is also assumed that all the $q+1$ intermediate curves have the same number of internal knots $(m+1)$. The complete knot sequence can be determined using equation (2). This produces generally satisfying values that reflect the distribution of the contour points. But some gaps in the knot sequence may appear when the points are unevenly spaced or the number of points is very different from a contour to another. We solve this problem by re-sampling adaptively new data points in these gaps.

Another requirement for the curve compatibility is the alignment of the given contours. Since a misalignment can produce twisted surfaces, the choice of a good alignment is fundamental for the quality of the resulting surface.

First of all, the contour points should be arranged in the same order, either clockwise or counterclockwise.

Next, the contours should be properly aligned. That means that a starting point should be found for each contour. The successive starting points form the longitudinal spine of the surface.

A good spine should be short in length and planar. We have developed an algorithm that computes the longitudinal spine according to the previous conditions, and taking into account the shape of the contours. The results of the choice of a misaligned and an aligned spine is shown on figure (1).



(a) Initial data (b) Twisted contours (c) Correct contours

Figure 1: Consequence of the choice of a longitudinal spine

3.3 Surface skinning

We assume here that all the $q+1$ curves are defined on the same knot vector, have a common degree k and $m+1$ distinct control points ordered counterclockwise. We want to construct a skinned surface from this set of closed B-spline curves [Woo88, PK96, PT96]. The resulting biparametric surface should be closed in the contours' direction and open in the longitudinal one. Equation (3) can be rewritten for each curve C_h:

$$C_h(u) = S(u, v_h) = \sum_{i=0}^{m+k-1}\left\{\sum_{j=0}^{n} Q_{i mod(m+1),j} N_j^l(v_h)\right\} N_{i mod(m+1)}^k(u)$$
$$\text{with } (u \in \mathbb{R}) \text{ and } (0 \leq v_h \leq 1)$$

To determine the points $Q_{i,j}$, representing the control net of the tensor-product surface, we fit an open B-spline curve to each column $(i = 0, \ldots, m)$ of the control points of the intermediate curves. By combining the previous equation with equation (1), we obtain:

$$P_{h,i} = \sum_{j=0}^{n} Q_{i,j} N_j^l(v_h) \qquad (h = 0, \ldots, q)$$

The determination of the control points of each longitudinal curve can be achieved by solving the matrix system $[P_i] = [N].[Q_i]$ and the final surface is obtained simply by joining the results together. The $[P_i]$ is a $(q+1)$ vector of the known data points representing the i^{th} control point on each section curve C_h, $[N]$ is the B-spline basis coefficient matrix of dimension $(q+1) \times (n+1)$, and $[Q_i]$ is the vector of dimension $(n+1)$ representing the unknown control points.

To solve these linear systems, we should assign a parameter to each data point $P_{h,i}$. To take into account the distribution of the points along the i^{th} longitudinal curve, we define a parameterization according to the chord-length method. To improve the final skinned surface, we determinate a common parameterization for all the columns by averaging the values:

$$\bar{t}_h = \frac{1}{m+1}\sum_{i=0}^{m} \bar{t}_h^i \qquad (h = 0, \ldots, q)$$

When $n = q$, the matrix $[N]$ is square and the $n+1$ control points can easily be obtained by $[Q_i] = [N]^{-1}.[P_i]$. This leads to the well-studied curve interpolation problem [RF89]. The N_i^k are defined on the common knot vector $\{u_i\}$ defined at section (3.2). The N_j^l should be defined on an appropriate knot vector $\{v_i\}$. To avoid any singularity in the system, we use the following technique of averaging the parameters:

$$v_0 = \ldots = v_l = 0 \text{ and } v_{n+1} = \ldots = v_{n+l+1} = 1$$

$$v_{j+l} = \frac{1}{l} \sum_{i=j}^{j+l-1} \overline{v}_i \qquad (j = 1, \ldots, n-l)$$

With this method the knots reflect the distribution of the $\overline{t}_i$.

When $n < q$, the matrix $[N]$ is no more square, the system should be solved by a least-square approximation method. This leads to a great amount of data reduction and is worth to be used when the number of cross-sections is important. Similar to the knot placement of section (3.1), we define a knot vector for the v direction:
let $d = (q+1)/(n+1)$, then

$$v_0 = \ldots = v_l = 0 \text{ and } v_{n+1} = \ldots = v_{n+l+1} = 1$$

$$v_{i+l} = (1-\alpha).\overline{t}_j + \alpha.\overline{t}_{j+1} \qquad (i = 1, \ldots, n-l)$$

where $j = int(id)$ and $\alpha = id - j$.

For both interpolating and approximating methods, the resulting smooth surface is C^{k-1} continuous in the u direction and C^{l-1} in the other direction.

3.4 Closing Surface

Since the initial data are structured as closed contours in one direction and open in the other direction, the isoparametric curves will consequently be closed in one direction and open in the other direction. However, this may be not satisfying for some anatomic shapes which are closed volumes, typically the bladder and prostate in our specific application.

One way to solve this problem is to cap the surface by a planar polygon but this leads to discontinuities in shape. Another way is to compute a smooth surface over the capping region and to ensure continuity with joining patches, as described in [Per92]. But this method requires sophisticated techniques, and moreover generates two additional patches.

Consequently, we have developed a simple solution to the closing problem. The idea is to extend the surface so that all the longitudinal curves converge to a same single point. This permits to model the whole object with a single B-spline surface. For this, we add a contour reduced to one point on top and bottom of the object, and we perform the previously described skinning method, on the new set of contours.

We first compute the gravity center of the two last (and first) polygonal contours G_{q-1} and G_q, and we take the symmetric of G_{q-1} about G_q, as shown on figure (2). Then, some constraints can be added at the chosen end-point to preserve the continuity without altering the shape of the reconstructed object. An example of closing is shown on figure (3).

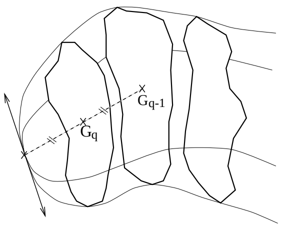


Figure 2: The closing method

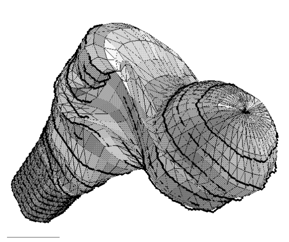
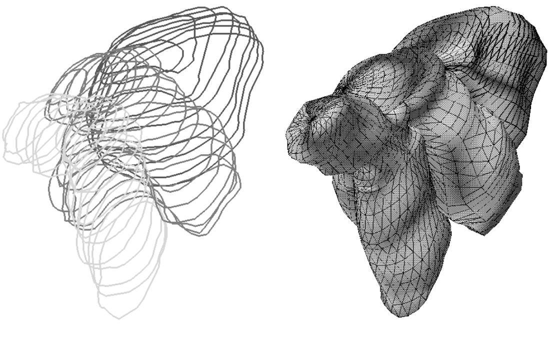


Figure 3: Closing of the femoral head

4 Results

The initial data are planar slices representing three internal neighbouring organs: rectum, prostate and bladder (Fig. 4(a)). We have reconstructed these objects with bicubic ($k = l = 3$) periodic B-spline surfaces with 15×10 control points. This has permitted to reduce dramatically the amount of the initial data, while producing C^2 continuous surfaces. A triangulated representation of the surfaces is shown in figure (4(b)).



(a) Initial data

(b) Periodic B-Spline surface models

Figure 4: Reconstruction of internal organs: rectum, prostate and bladder

We have also reconstructed lungs from particularly disturbed contours containing approximatively 5000 points (Fig. 5). The lungs have been modeled with periodic biquadratic B-spline surfaces with 30x15 control points. The results on figure (6) show the quality of the resulting surface.

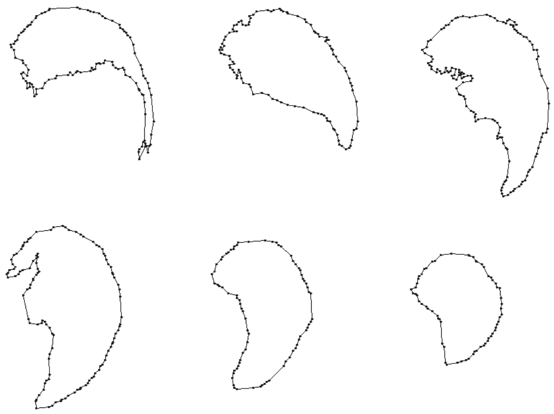


Figure 5: Some initial contours of the left lung

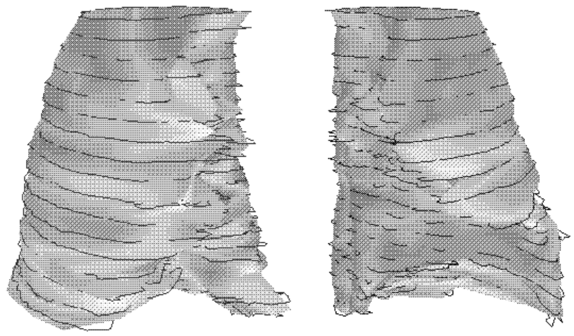


Figure 6: Initial contours and reconstructed surface of lungs

5 Conclusion

In this paper, an algorithm for the skinning of closed surfaces has been presented. The method generates quickly smooth surfaces with high continuity from planar closed contours. Interpolating can produce oscillating surface when successive contours have very different shapes. Thus, both the contour curves and the skinned surface are approximated, this leads to an important data reduction.

The methods proposed in previous research works [PK96, PT96] for curves compatibility, generate a great number of control points due to the accuracy and the smoothness requirements. This is normally followed by a very time consuming phase of reducing the number of knots of the common knot vector (about 90% of the total skinning time). Thus, we have proposed a simple averaging method to define the section curves on a common knot vector. The produced surface fit well the initial data, while keeping a small number of control points, and the physicians have been satisfied of the accuracy of the proposed model.

We have also proposed to close the surface by adding new contours to the top and bottom of the object. However, this method may create distorted surfaces as soon as the extremity contours are no more convex (or quasi convex). Thus we are currently working on extending the surface into an extremity curve whose shape is close to the planar threadlike skeleton of the last contour, rather than into a single point contour as proposed in section (3.4). But many continuity and curve compatibility problems arise.

Though, in this paper, we have studied the reconstruction of sections containing only one contour, it is feasible to modify the method in order to handle the case of multiple contours. Once we have solved the correspondence problem of the contours in adjacent sections, we reconstruct each branch separately. Then, the branches can be linked together with joining patches. We can use the techniques described in [Per92], in order to join the capping regions to the skinned surfaces. However, the case with many branches (four and more) is very complex and the continuity is difficult to ensure everywhere. The general case remains an open research area.

Finally, the proposed skinning method seems to be adequate and efficient enough to be integrated in an anatomical oriented interface (Fig. 7). As the precision of the reconstruction is not the main requirement, our application produces, at interactive time, realistic models of 3D closed shapes. These help visually the physicians during the radiotherapy treatment planning or iso-dose calculation.

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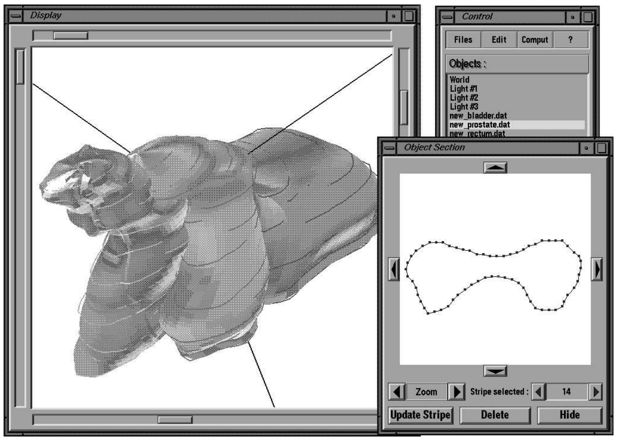

Figure 7: The interface

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