



2016 RIVA DEL GARDA

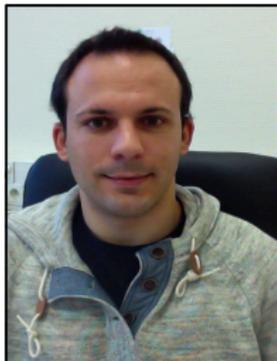


# Preference-based Pattern Mining

Bruno Crémilleux, Marc Plantevit, Arnaud Soulet

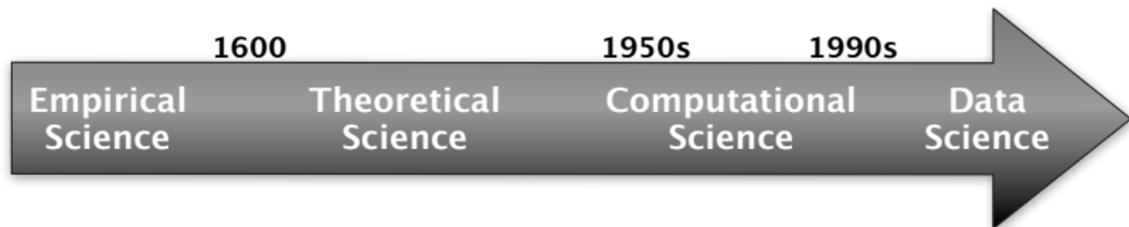
**Riva del Garda, Italy - September 19, 2016**

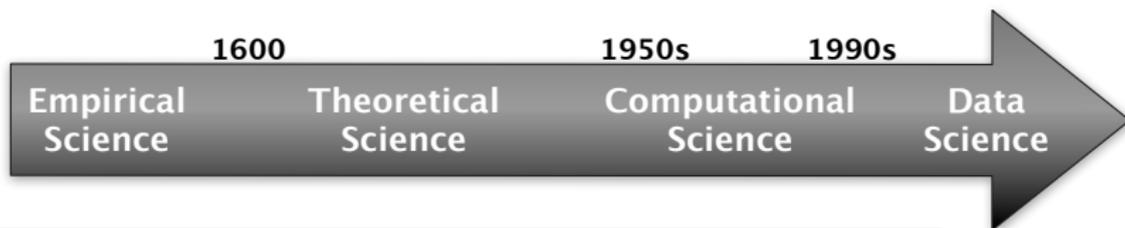
# Introduction



- Bruno Crémilleux, Professor, Univ. Caen, France.
- Marc Plantevit, Associate Professor, Univ. Lyon, France.
- Arnaud Soulet, Associate Professor, Univ. Tours, France.

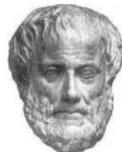
Material available on <http://liris.cnrs.fr/~mplantev/doku/doku.php?id=preferencebasedpatternminingtutorial>





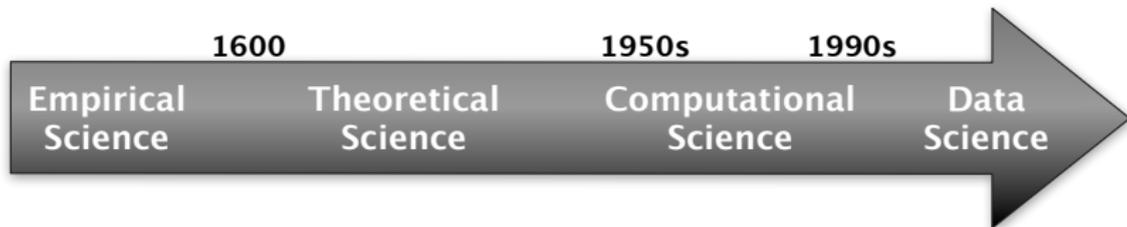
## Before 1600: Empirical Science

- Babylonian mathematics: 4 basis operations done with tablets and the resolution of practical problems based on words describing all the steps.  $\Rightarrow$  able to solve 3-degree equations.
- Ancient Egypt: No theorization of algorithms. Only examples made empirically, certainly repeated by students and scribes. Empirical knowledge transmitted as such and not a rational mathematical science.
- Aristotle also produced many biological writings that were empirical in nature, focusing on biological causation and the diversity of life. He made countless observations of nature, especially the habits and attributes of plants and animals in the world around him, classified more than 540 animal species, and dissected at least 50.
- ...



Wikipedia

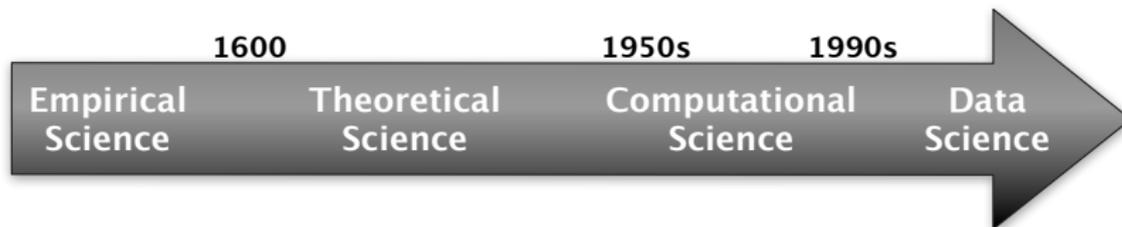




## 1950s–1990s, Computational Science

- Over the last 50 years, most disciplines have grown a third, computational branch (e.g. empirical, theoretical, and computational ecology, or physics, or linguistics.)
- Computational Science traditionally meant simulation. It grew out of our inability to find closed form solutions for complex mathematical models.





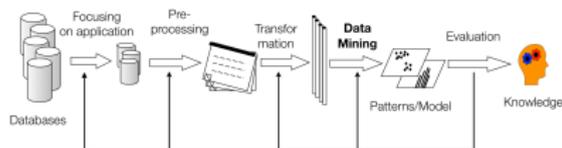
## 1990's-now, the Data Science Era

- The flood of data from new scientific instruments and simulations
- The ability to economically store and manage petabytes of data online
- The Internet and computing Grid that makes all these archives universally accessible
- Scientific info. management, acquisition, organization, query, and visualization tasks scale almost linearly with data volumes.

## The Fourth Paradigm: Data-Intensive Scientific Discovery

Data mining is a major new challenge!





Iterative and Interactive Process

## Data Mining

- Core of KDD
- Search for knowledge in data

 Fayad et al., 1996

## Functionalities

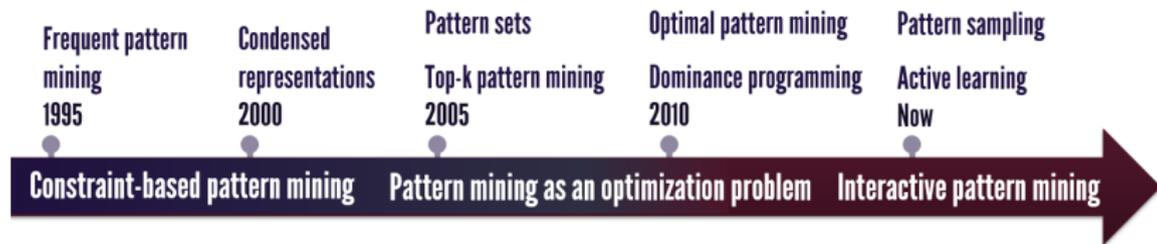
- **Descriptive data mining** vs Predictive data mining
- **Pattern mining**, classification, clustering, regression
- Characterization, discrimination, association, classification, clustering, outlier and trend analysis, etc.

We will focus on **descriptive data mining** especially on Constraint-based Pattern Mining with an **inductive database vision**.

$$Th(\mathcal{L}, \mathcal{D}, \mathcal{C}) = \{\psi \in \mathcal{L} \mid \mathcal{C}(\psi, \mathcal{D}) \text{ is true}\}$$

- Pattern domain: itemset, sequences, graphs, dynamic graphs, etc.
- Constraints (frequency, area, statistical relevancy, cliqueness, etc.): How to efficiently push them?

 Imielinski and Mannila: Communications of the ACM (1996).



How have we moved from (only) frequent pattern discovery to interactive pattern mining?

How have we moved from the retrieval era to the exploratory analysis era?

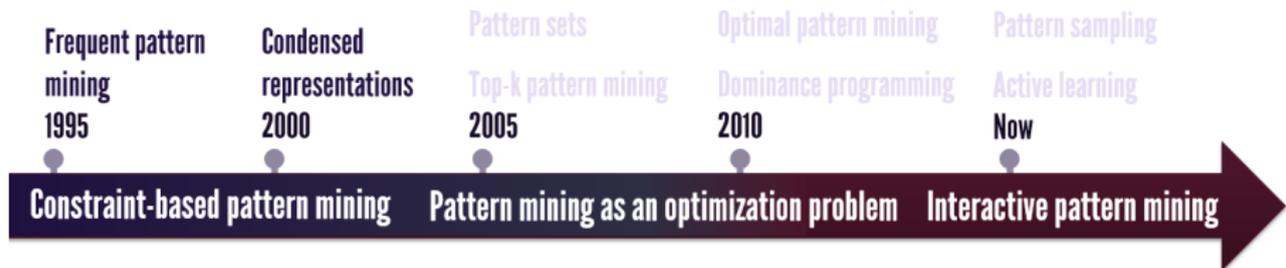
- A very short view on the constraint-based pattern mining toolbox and its limitation 
- Claim #1: this is not a tutorial on constraint-based pattern mining!

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- Pattern mining as an optimization problem based on user's preferences: 
  - From all solutions to the optimal ones (top  $k$ , skyline, pattern set, etc.).
  - Claim #2: this is not a tutorial on preference learning!

- A very short view on the constraint-based pattern mining toolbox and its limitation 
  - **Claim #1: this is not a tutorial on constraint-based pattern mining!**
- Pattern mining as an optimization problem based on user's preferences: 
  - From all solutions to the optimal ones (top  $k$ , skyline, pattern set, etc.).
  - **Claim #2: this is not a tutorial on preference learning!**
- Interactive pattern mining: 
  - Dealing with implicit user's preferences.
  - How to ensure interactivity (instant mining, pattern space sampling)
  - Forgetting the completeness of the extraction.
  - **Claim #3: this is not a tutorial on preference learning either!**



- We have done some enlightenment choices.
  - Linearisation of the pattern mining research history.
- We are not exhaustive !
  - Feel free to mention us some important papers that are missing.
- Most of the examples will consider the itemsets as pattern language.
  - It is the simplest to convey the main ideas and intuitions.
- Feel free to interrupt us at any time if you have some questions.



## Constraint-based pattern mining: the toolbox and its limits

➡ the need of preferences in pattern mining

## Definition

Given a set of attributes  $\mathcal{A}$ , an itemset  $X$  is a subset of attributes, i. e.,  $X \subseteq \mathcal{A}$ .

Input:

	$a_1$	$a_2$	$\dots$	$a_n$
$o_1$	$d_{1,1}$	$d_{1,2}$	$\dots$	$d_{1,n}$
$o_2$	$d_{2,1}$	$d_{2,2}$	$\dots$	$d_{2,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$o_m$	$d_{m,1}$	$d_{m,2}$	$\dots$	$d_{m,n}$

## Question

How many itemsets are there?  
 $2^{|\mathcal{A}|}$ .

where  $d_{i,j} \in \{\text{true}, \text{false}\}$

# Transactional representation of the data

Relational representation:  $\mathcal{D} \subseteq \mathcal{O} \times \mathcal{A}$

	$a_1$	$a_2$	$\dots$	$a_n$
$o_1$	$d_{1,1}$	$d_{1,2}$	$\dots$	$d_{1,n}$
$o_2$	$d_{2,1}$	$d_{2,2}$	$\dots$	$d_{2,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$o_m$	$d_{m,1}$	$d_{m,2}$	$\dots$	$d_{m,n}$

where  $d_{i,j} \in \{\text{true}, \text{false}\}$

Transactional representation:  $\mathcal{D}$  is an array of subsets of  $\mathcal{A}$

$t_1$   
 $t_2$   
 $\vdots$   
 $t_m$

where  $t_i \subseteq \mathcal{A}$

## Example

	$a_1$	$a_2$	$a_3$
$o_1$	x	x	x
$o_2$	x	x	
$o_3$		x	
$o_4$			x

$t_1$	$a_1, a_2, a_3$
$t_2$	$a_1, a_2$
$t_3$	$a_2$
$t_4$	$a_3$

## Definition (absolute frequency)

Given the objects in  $\mathcal{O}$  described with the Boolean attributes in  $\mathcal{A}$ , the absolute frequency of an itemset  $X \subseteq \mathcal{A}$  in the dataset  $\mathcal{D} \subseteq \mathcal{O} \times \mathcal{A}$  is  $|\{o \in \mathcal{O} \mid \{o\} \times X \subseteq \mathcal{D}\}|$ .

## Definition (relative frequency)

Given the objects in  $\mathcal{O}$  described with the Boolean attributes in  $\mathcal{A}$ , the relative frequency of an itemset  $X \subseteq \mathcal{A}$  in the dataset  $\mathcal{D} \subseteq \mathcal{O} \times \mathcal{A}$  is  $\frac{|\{o \in \mathcal{O} \mid \{o\} \times X \subseteq \mathcal{D}\}|}{|\mathcal{O}|}$ .

The relative frequency is a joint probability.

## Problem Definition

Given the objects in  $\mathcal{O}$  described with the Boolean attributes in  $\mathcal{A}$ , listing every itemset having a frequency above a given threshold  $\mu \in \mathbb{N}$ .

Input:

	$a_1$	$a_2$	$\dots$	$a_n$
$o_1$	$d_{1,1}$	$d_{1,2}$	$\dots$	$d_{1,n}$
$o_2$	$d_{2,1}$	$d_{2,2}$	$\dots$	$d_{2,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$o_m$	$d_{m,1}$	$d_{m,2}$	$\dots$	$d_{m,n}$

and a minimal frequency  $\mu \in \mathbb{N}$ .

where  $d_{i,j} \in \{\text{true}, \text{false}\}$



R. Agrawal; T. Imielinski; A. Swami: Mining Association Rules Between Sets of Items in Large Databases, SIGMOD, 1993.

## Problem Definition

Given the objects in  $\mathcal{O}$  described with the Boolean attributes in  $\mathcal{A}$ , listing every itemset having a frequency above a given threshold  $\mu \in \mathbb{N}$ .

Output: every  $X \subseteq \mathcal{A}$  such that there are at least  $\mu$  objects having all attributes in  $X$ .



R. Agrawal; T. Imielinski; A. Swami: Mining Association Rules Between Sets of Items in Large Databases, SIGMOD, 1993.

Specifying a minimal absolute frequency  $\mu = 2$  objects (or, equivalently, a minimal relative frequency of 50%).

	$a_1$	$a_2$	$a_3$
$o_1$	×	×	×
$o_2$	×	×	
$o_3$		×	
$o_4$			×

Specifying a minimal absolute frequency  $\mu = 2$  objects (or, equivalently, a minimal relative frequency of 50%).

	$a_1$	$a_2$	$a_3$
$o_1$	×	×	×
$o_2$	×	×	
$o_3$		×	
$o_4$			×

The frequent itemsets are:  $\emptyset$  (4),  $\{a_1\}$  (2),  $\{a_2\}$  (3),  $\{a_3\}$  (2) and  $\{a_1, a_2\}$  (2).

Querying data:

$$\{d \in \mathcal{D} \mid q(d, \mathcal{D})\}$$

where:

- $\mathcal{D}$  is a dataset (tuples),
- $q$  is a query.

Querying patterns:

$$\{X \in P \mid Q(X, \mathcal{D})\}$$

where:

- $\mathcal{D}$  is the dataset,
- $P$  is the pattern space,
- $Q$  is an inductive query.

Querying **the frequent itemsets**:

$$\{X \in P \mid Q(X, \mathcal{D})\}$$

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- $\mathcal{D}$  is the dataset,
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Querying **the frequent itemsets**:

$$\{X \in P \mid Q(X, \mathcal{D})\}$$

where:

- $\mathcal{D}$  is a subset of  $\mathcal{O} \times \mathcal{A}$ , i. e., objects described with Boolean attributes,
- $P$  is the pattern space,
- $Q$  is an inductive query.

Querying **the frequent itemsets**:

$$\{X \in P \mid Q(X, \mathcal{D})\}$$

where:

- $\mathcal{D}$  is a subset of  $\mathcal{O} \times \mathcal{A}$ , i. e., objects described with Boolean attributes,
- $P$  is  $2^{\mathcal{A}}$ ,
- $Q$  is an inductive query.

Querying **the frequent itemsets**:

$$\{X \in P \mid Q(X, \mathcal{D})\}$$

where:

- $\mathcal{D}$  is a subset of  $\mathcal{O} \times \mathcal{A}$ , i. e., objects described with Boolean attributes,
- $P$  is  $2^{\mathcal{A}}$ ,
- $Q$  is  $(X, \mathcal{D}) \mapsto |\{o \in \mathcal{O} \mid \{o\} \times X \subseteq \mathcal{D}\}| \geq \mu$ .

Querying **the frequent itemsets**:

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Querying **the frequent itemsets**:

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- $P$  is  $2^{\mathcal{A}}$ ,
- $Q$  is  $(X, \mathcal{D}) \mapsto f(X, \mathcal{D}) \geq \mu$ .

Listing the frequent itemsets is NP-hard.

$$\mu = 2$$

$\mathcal{O}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
$\mathcal{O}_1$	x	x	x	x	x										
$\mathcal{O}_2$	x	x	x	x	x										
$\mathcal{O}_3$	x	x	x	x	x										
$\mathcal{O}_4$						x	x	x	x	x					
$\mathcal{O}_5$						x	x	x	x	x					
$\mathcal{O}_6$						x	x	x	x	x					
$\mathcal{O}_7$											x	x	x	x	x
$\mathcal{O}_8$											x	x	x	x	x

- How many frequent patterns?

$$\mu = 2$$

$\mathcal{O}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
$\mathcal{O}_1$	x	x	x	x	x										
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$\mathcal{O}_5$						x	x	x	x	x					
$\mathcal{O}_6$						x	x	x	x	x					
$\mathcal{O}_7$											x	x	x	x	x
$\mathcal{O}_8$											x	x	x	x	x

- How many frequent patterns?  $1 + (2^5 - 1) \times 3 = 94$  patterns

$$\mu = 2$$

$\mathcal{O}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
$\mathcal{O}_1$	x	x	x	x	x										
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$\mathcal{O}_7$											x	x	x	x	x
$\mathcal{O}_8$											x	x	x	x	x

- How many frequent patterns?  $1 + (2^5 - 1) \times 3 = 94$  patterns but actually 4 (potentially) interesting ones:  $\{\}, \{a_1, a_2, a_3, a_4, a_5\}, \{a_6, a_7, a_8, a_9, a_{10}\}, \{a_{11}, a_{12}, a_{13}, a_{14}, a_{15}\}$ .

$$\mu = 2$$

$\mathcal{O}$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
$\mathcal{O}_1$	x	x	x	x	x										
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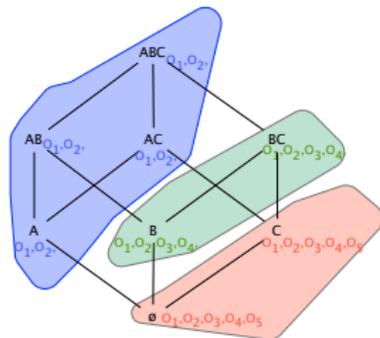
☞ the need to focus on a **condensed representation** of frequent patterns.



Toon Calders, Christophe Rigotti, Jean-François Boulicaut: A Survey on Condensed Representations for Frequent Sets. Constraint-Based Mining and Inductive Databases 2004: 64-80.

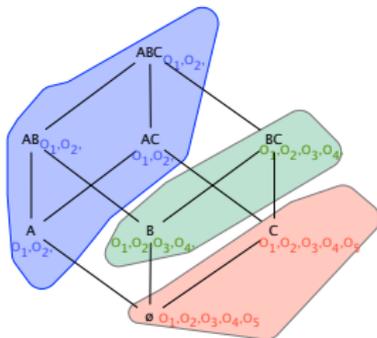
Equivalence classes based on support.

$\mathcal{O}$	A	B	C
$\mathcal{O}_1$	×	×	×
$\mathcal{O}_2$	×	×	×
$\mathcal{O}_3$		×	×
$\mathcal{O}_4$		×	×
$\mathcal{O}_5$			×



Equivalence classes based on support.

$\mathcal{O}$	A	B	C
$o_1$	×	×	×
$o_2$	×	×	×
$o_3$		×	×
$o_4$		×	×
$o_5$			×



- **Closed** patterns are maximal element of each equivalence class (Bastide et al., SIGKDD Exp. 2000):  $ABC$ ,  $BC$ , and  $C$ .
- **Generators** or **Free** patterns are minimal elements (not necessary unique) of each equivalent class (Boulicaut et al, DAMI 2003):  $\{\}$ ,  $A$  and  $B$

A strong intersection with Formal Concept Analysis (Ganter and Wille, 1999).



## Constraints are needed for:

- only retrieving patterns that describe an interesting subgroup of the data
- making the extraction feasible

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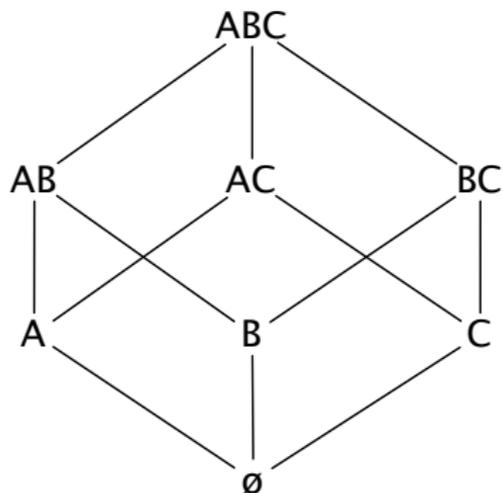
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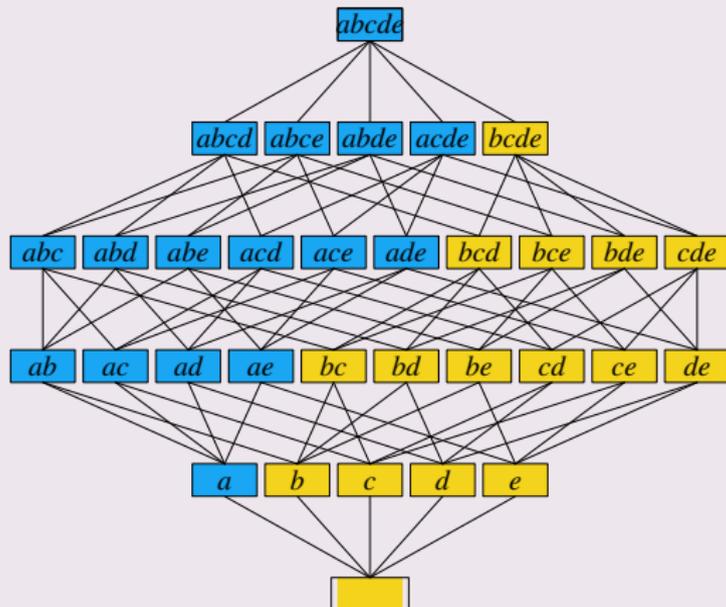
- They are defined up to the partial order  $\preceq$  used for listing the patterns



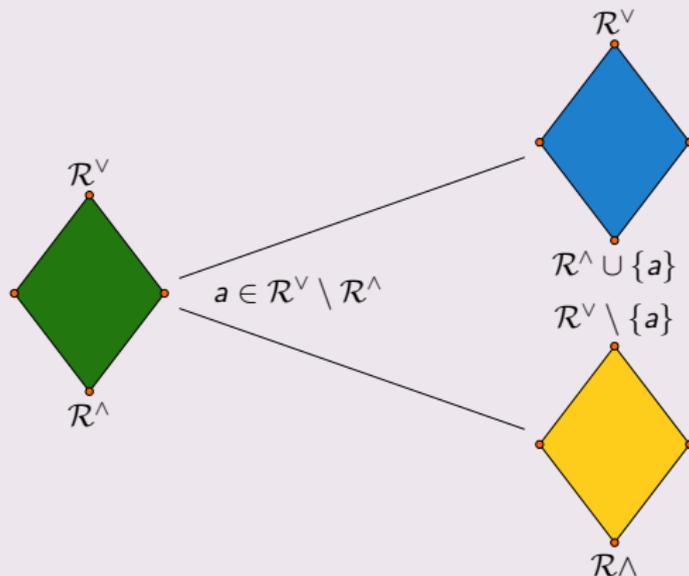
Levelwise enumeration vs  
depth-first enumeration.

Whatever the enumeration principles, we have to derive some pruning properties from the constraints.

Binary partition: the element 'a' is enumerated



## Binary partition: the element 'a' is enumerated



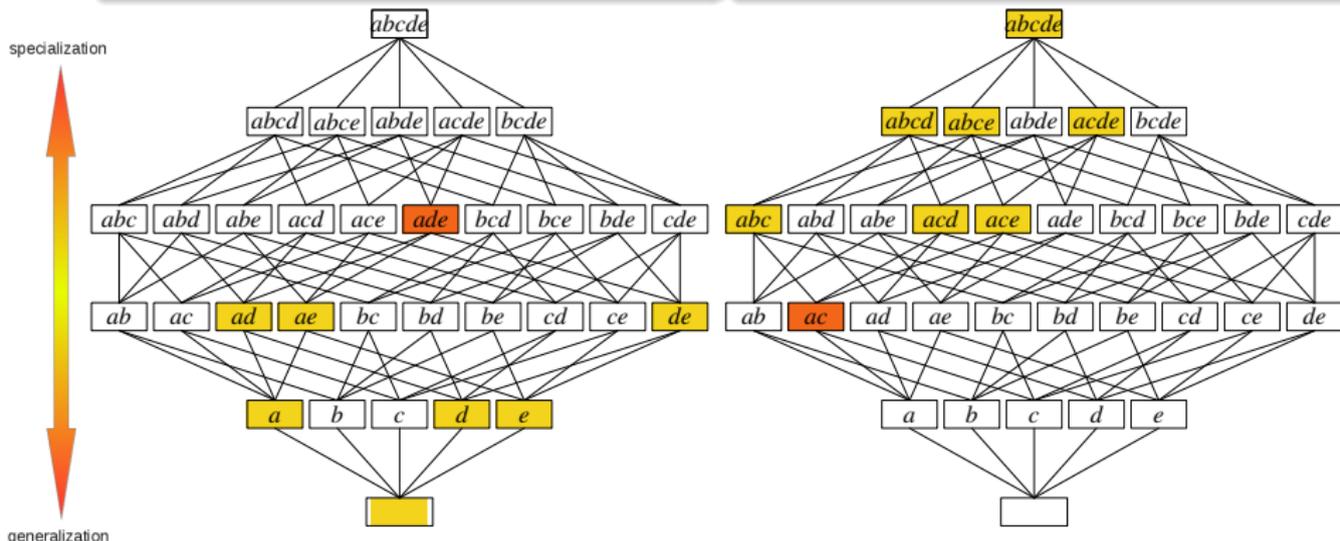
# (Anti-)Monotone Constraints

## Monotone constraint

$$\forall \varphi_1 \preceq \varphi_2, \mathcal{C}(\varphi_1, \mathcal{D}) \Rightarrow \mathcal{C}(\varphi_2, \mathcal{D})$$

## Anti-monotone constraint

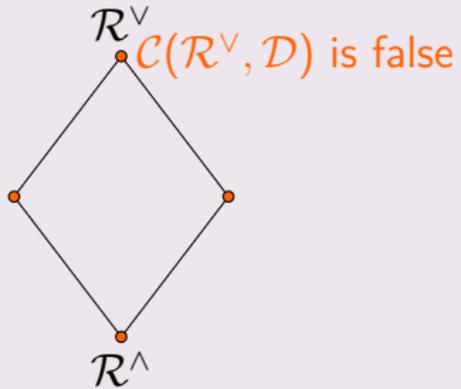
$$\forall \varphi_1 \preceq \varphi_2, \mathcal{C}(\varphi_2, \mathcal{D}) \Rightarrow \mathcal{C}(\varphi_1, \mathcal{D})$$



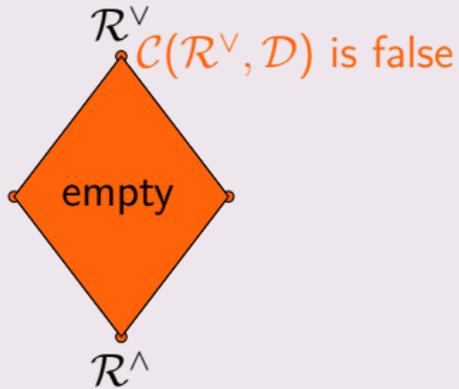
$$\mathcal{C}(\varphi, \mathcal{D}) \equiv b \in \varphi \vee c \in \varphi$$

$$\mathcal{C}(\varphi, \mathcal{D}) \equiv a \notin \varphi \wedge c \notin \varphi$$

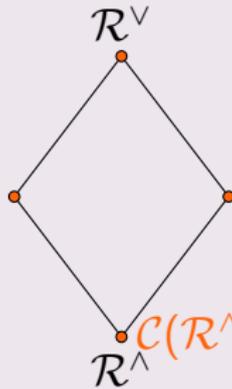
## Monotone constraint



## Monotone constraint

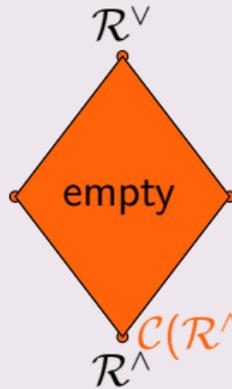


## Anti-monotone constraint



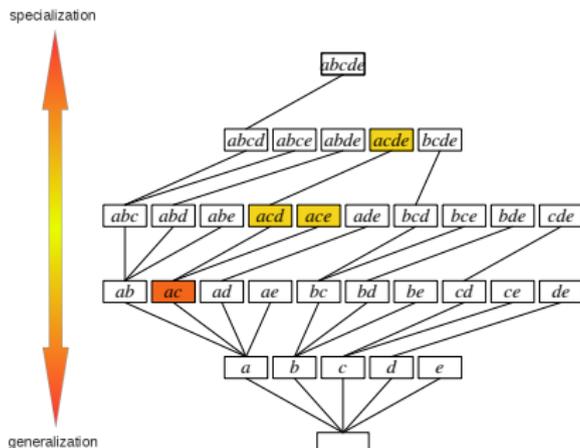
$C(\mathcal{R}^A, \mathcal{D})$  is false

## Anti-monotone constraint



## Convertible constraints (Pei et al., DAMI 2004)

$\preceq$  is extended to the prefix order  $\leq$  so that  $\forall \varphi_1 \leq \varphi_2, \mathcal{C}(\varphi_2, \mathcal{D}) \Rightarrow \mathcal{C}(\varphi_1, \mathcal{D})$

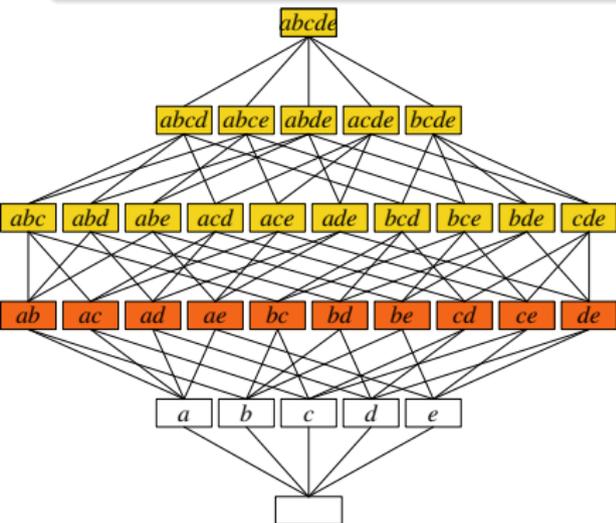


$$\mathcal{C}(\varphi, w) \equiv \text{avg}(w(\varphi)) > \sigma$$

$$w(a) \geq w(b) \geq w(c) \geq w(d) \geq w(e)$$

## Loose AM constraints

$$\mathcal{C}(\varphi, \mathcal{D}) \Rightarrow \exists e \in \varphi : \mathcal{C}(\varphi \setminus \{e\}, \mathcal{D})$$



$$\mathcal{C}(\varphi, w) \equiv \text{var}(w(\varphi)) \leq \sigma$$

 Bonchi and Lucchese – DKE 2007

 Uno, ISAAC07

$v \in P$	M
$P \supseteq S$	M
$P \subseteq S$	AM
$\min(P) \leq \sigma$	AM
$\min(P) \geq \sigma$	M
$\max(P) \leq \sigma$	M
$\max(P) \geq \sigma$	AM
$\text{range}(P) \leq \sigma$	AM
$\text{range}(P) \geq \sigma$	M
$\text{avg}(P)\theta\sigma, \theta \in \{\leq, =, \geq\}$	Convertible
$\text{var}(w(\varphi)) \leq \sigma$	LAM

Some constraints can be decomposed into several pieces that are either monotone or anti-monotone.

- Piecewise monotone and anti-monotone constraints

 L. Cerf, J. Besson, C. Robardet, J-F. Boulicaut: Closed patterns meet n-ary relations. TKDD 3(1) (2009)

- Primitive-based constraints

 A.Soulet, B. Crémilleux: Mining constraint-based patterns using automatic relaxation. Intell. Data Anal. 13(1): 109-133 (2009)

- Projection-antimonotonicity

 A. Buzmakov, S. O. Kuznetsov, A.Napoli: Fast Generation of Best Interval Patterns for Nonmonotonic Constraints. ECML/PKDD (2) 2015: 157-172

- $\forall e, w(e) \geq 0$
- $\mathcal{C}(\varphi, w) \equiv \text{avg}(w(\varphi)) > \sigma \equiv \frac{\sum_{e \in \varphi} w(e)}{|\varphi|} > \sigma.$

$\mathcal{C}(\varphi, \mathcal{D})$  is piecewise monotone and anti-monotone with

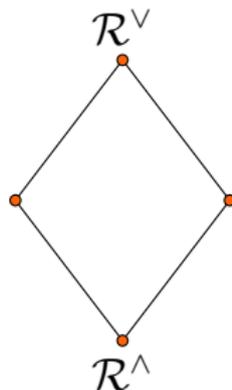
$$f(\varphi_1, \varphi_2, \mathcal{D}) = \frac{\sum_{e \in \varphi_1} w(e)}{|\varphi_2|}$$

$\forall x \preceq y,$

- $f_{1,\varphi}$  is monotone:  $f(x, \varphi_2, \mathcal{D}) = \frac{\sum_{e \in x} w(e)}{|\varphi_2|} > \sigma \Rightarrow \frac{\sum_{e \in y} w(e)}{|\varphi_2|} > \sigma$
- $f_{2,\varphi}$  is anti-monotone:  
 $f(\varphi_1, y, \mathcal{D}) = \frac{\sum_{e \in \varphi_1} w(e)}{|y|} > \sigma \Rightarrow \frac{\sum_{e \in \varphi_1} w(e)}{|x|} > \sigma$

## Evaluation

$$\text{If } f(\mathcal{R}^{\vee}, \mathcal{R}^{\wedge}, \mathcal{D}) = \frac{\sum_{e \in \mathcal{R}^{\vee}} w(e)}{|\mathcal{R}^{\wedge}|}$$

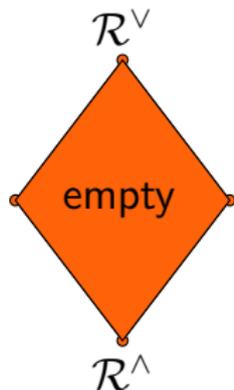


## Propagation

- $\exists e \in \mathcal{R}^{\vee} \setminus \mathcal{R}^{\wedge}$  such that  $f(\mathcal{R}^{\vee} \setminus \{e\}, \mathcal{R}^{\wedge}, \mathcal{D}) \leq \sigma$ , then  $e$  is moved in  $\mathcal{R}^{\wedge}$
- $\exists e \in \mathcal{R}^{\vee} \setminus \mathcal{R}^{\wedge}$  such that  $f(\mathcal{R}^{\vee}, \mathcal{R}^{\wedge} \cup \{e\}, \mathcal{D}) \leq \sigma$ , then  $e$  is removed from  $\mathcal{R}^{\vee}$

## Evaluation

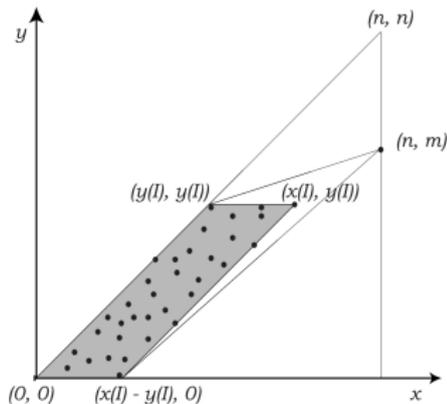
If  $f(\mathcal{R}^\vee, \mathcal{R}^\wedge, \mathcal{D}) = \frac{\sum_{e \in \mathcal{R}^\vee} w(e)}{|\mathcal{R}^\wedge|}$   
 $\leq \sigma$  then  $\mathcal{R}$  is empty.



## Propagation

- $\exists e \in \mathcal{R}^\vee \setminus \mathcal{R}^\wedge$  such that  $f(\mathcal{R}^\vee \setminus \{e\}, \mathcal{R}^\wedge, \mathcal{D}) \leq \sigma$ , then  $e$  is moved in  $\mathcal{R}^\wedge$
- $\exists e \in \mathcal{R}^\vee \setminus \mathcal{R}^\wedge$  such that  $f(\mathcal{R}^\vee, \mathcal{R}^\wedge \cup \{e\}, \mathcal{D}) \leq \sigma$ , then  $e$  is removed from  $\mathcal{R}^\vee$

- Convex measures can be taken into account by computing some upper bounds with  $\mathcal{R}^\wedge$  and  $\mathcal{R}^\vee$ .
- Branch and bound enumeration



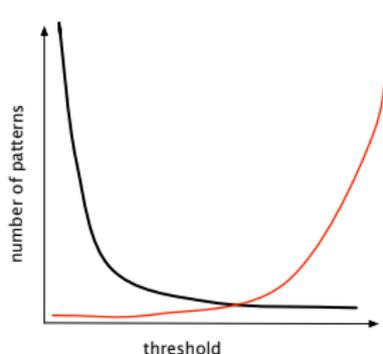
 Shinichi Morishita, Jun Sese: Traversing Itemset Lattice with Statistical Metric Pruning. PODS 2000: 226-236

## Why declarative approaches?

- for each problem, do not write a solution from scratch

## Declarative approaches:

- CP approaches (Khiari et al., CP10, Guns et al., TKDE 2013)
- SAT approaches (Boudane et al., IJCAI16, Jabbour et al., CIKM13)
- ILP approaches (Mueller et al, DS10, Babaki et al., CPAIOR14, Ouali et al. IJCAI16)
- ASP approaches (Gebser et al., IJCAI16)

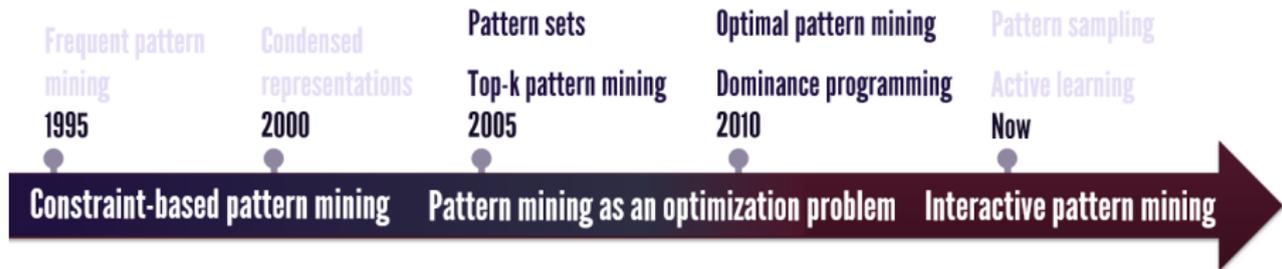


- A too stringent threshold: trivial patterns
- A too weak threshold: too many patterns, unmanageable and diversity not necessary assured.
- Some attempts to tackle this issue:
  - Interestingness is not a dichotomy! [BB05]
  - Taking benefit from hierarchical relationships [HF99, DPRB14]
- But setting thresholds remains an issue in pattern mining.

# Constraint-based pattern mining: concluding remarks

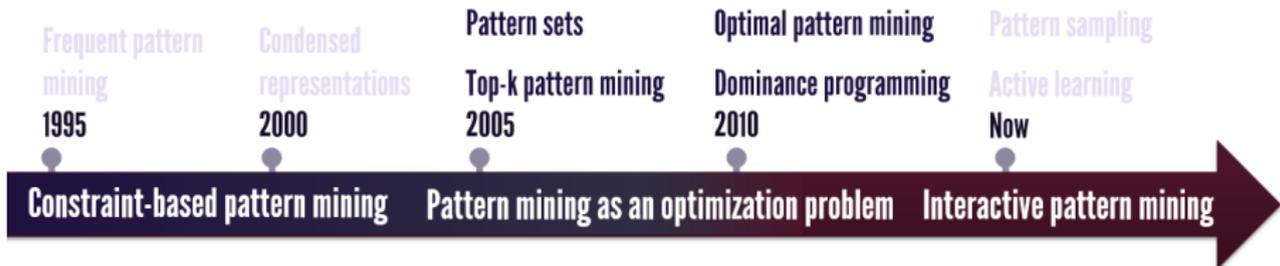
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- how to fix thresholds?
- how to handle numerous patterns including non-informative patterns? how to get a global picture of the set of patterns?
- how to design the proper constraints/preferences?



## Pattern mining as an optimization problem

# Pattern mining as an optimization problem



- performance issue
- the more, the better
- data-driven
- quality issue
- the less, the better
- user-driven

## In this part:

- preferences to express user's interests
- focusing on the best patterns: dominance relation, pattern sets, subjective interest

# Addressing pattern mining tasks with user preferences

**Idea:** a **preference** expresses a user's interest  
(no required threshold)

Examples based on **measures/dominance relation**:

- *“the higher the frequency, growth rate and aromaticity are, the better the patterns”*
- *“I prefer pattern  $X_1$  to pattern  $X_2$  if  $X_1$  is not dominated by  $X_2$  according to a set of measures”*

➔ measures/preferences: a natural criterion for ranking patterns and presenting the “best” patterns

# Preference-based approaches in this tutorial

---

- **in this part:** preferences are **explicit** (typically given by the user depending on his/her interest/subjectivity)  
**in the last part:** preferences are **implicit**
- *quantitative/qualitative preferences:*
  - **quantitative:**  
*measures*  $\left\{ \begin{array}{l} \textit{constraint-based data mining: frequency, size, \dots} \\ \textit{background knowledge: price, weight, aromaticity, \dots} \\ \textit{statistics: entropy, pvalue, \dots} \end{array} \right.$
  - **qualitative:** “I prefer pattern  $X_1$  to pattern  $X_2$ ” (pairwise comparison between patterns).  
With qualitative preferences: **two patterns can be incomparable.**

Many works on:

- **interestingness measures** (Geng et al. ACM Computing Surveys06)
- **utility functions** (Yao and Hamilton DKE06)
- **statistically significant rules** (Hämäläinen and Nykänen ICDM08)

**Examples:**

- $area(X) = frequency(X) \times size(X)$  (tiling: **surface**)

$$lift(X_1 \rightarrow X_2) = \frac{D \times frequency(X_1 X_2)}{frequency(X_2) \times frequency(X_1)}$$

- *utility functions*: utility of the mined patterns (e.g. weighted items, weighted transactions)

# Putting the pattern mining task to an optimization problem

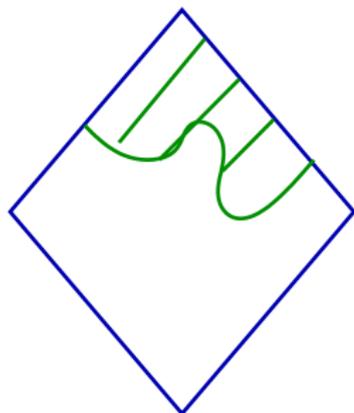
The most interesting patterns according to measures/preferences:

- **free/closed patterns** (Boulicaut et al. DAMI03, Bastide et al. SIGKDD Explorations00)
  - ➔ given an equivalent class, I prefer the shortest/longest patterns
- **one measure: top-k patterns** (Fu et al. Ismis00, Jabbour et al. ECML/PKDD13)
- **several measures**: how to find a trade-off between several criteria?
  - ➔ **skyline patterns** (Cho et al. IJDWM05, Soulet et al. ICDM'11, van Leeuwen and Ukkonen ECML/PKDD13)
- **dominance programming** (Negrevergne et al. ICDM13), **optimal patterns** (Ugarte et al. ICTAI15)
- **subjective interest/interest according to a background knowledge** (De Bie DAMI2011)

Goal: finding the k patterns maximizing an interestingness measure.

Tid	Items					
$t_1$		B		E	F	
$t_2$		B	C	D		
$t_3$	A			E	F	
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F

- the 3 most frequent patterns:  
 $B$ ,  $E$ ,  $BE^a$   
➔ easy due to the anti-monotone property of frequency

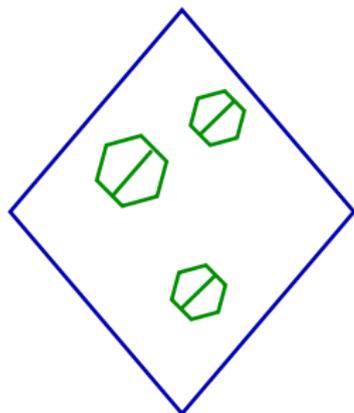


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<sup>a</sup>Other patterns have a frequency of 5:  
 $C$ ,  $D$ ,  $BC$ ,  $BD$ ,  $CD$ ,  $BCD$

Goal: finding the k patterns maximizing an interestingness measure.

Tid	Items					
$t_1$		B		E	F	
$t_2$		B	C	D		
$t_3$	A			E	F	
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



- the 3 most frequent patterns:  
 $B$ ,  $E$ ,  $BE^a$   
→ easy due to the anti-monotone property of frequency

- the 3 patterns maximizing area:  
 $BCDE$ ,  $BCD$ ,  $CDE$   
→ branch & bound  
(Zimmermann and De Raedt MLJ09)

---

<sup>a</sup>Other patterns have a frequency of 5:  
 $C$ ,  $D$ ,  $BC$ ,  $BD$ ,  $CD$ ,  $BCD$

# top- $k$ pattern mining

## an example of pruning condition

top- $k$  patterns according to *area*,  $k = 3$

Tid	Items					
$t_1$		B			E	F
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F

### Principle:

- *Cand*: the current set of the  $k$  best candidate patterns
- when a candidate pattern is inserted in *Cand*, a more efficient pruning condition is deduced

*A*: lowest value of *area* for the patterns in *Cand*

*L*: size of the longest transaction in  $\mathcal{D}$  (here:  $L = 6$ )

a pattern  $X$  must satisfy  $frequency(X) \geq \frac{A}{L}$  to be inserted in *Cand*

➡ pruning condition according to the frequency (thus anti-monotone)

Example with a depth first search approach:

- initialization:  $Cand = \{B, BE, BEC\}$   
( $area(BEC) = 12$ ,  $area(BE) = 10$ ,  $area(B) = 6$ )  
➡  $frequency(X) \geq \frac{6}{6}$
- new candidate  $BECD$ :  $Cand = \{BE, BEC, BECD\}$   
( $area(BECD) = 16$ ,  $area(BEC) = 12$ ,  $area(BE) = 10$ )  
➡  $frequency(X) \geq \frac{10}{6}$  which is more efficient than  $frequency(X) \geq \frac{6}{6}$
- new candidate  $BECDF \dots$

## Advantages:

- compact
- threshold free
- best patterns

## Drawbacks:

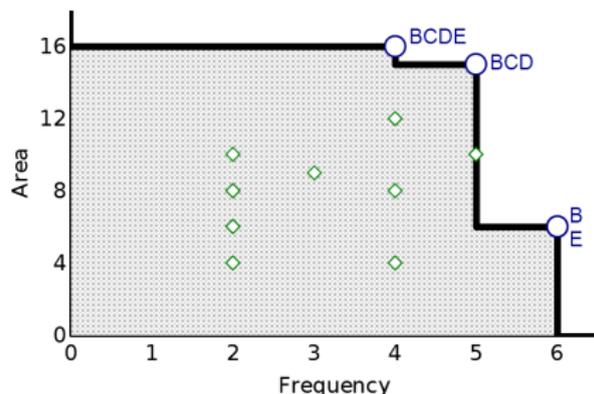
- complete resolution is costly, sometimes heuristic search (beam search)  
(van Leeuwen and Knobbe DAMI12)
- **diversity issue**: top- $k$  patterns are often very similar
- several criteria must be aggregated  
↳ **skylines patterns**: a trade-off between several criteria

# Skypatterns (Pareto dominance)

Notion of **skylines (database) in pattern mining** (Cho et al. IJDWM05, Papadopoulos et al. DAMI08, Soulet et al. ICDM11, van Leeuwen and Ukkonen ECML/PKDD13)

Tid	Items					
$t_1$		B		E	F	
$t_2$		B	C	D		
$t_3$	A			E	F	
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F

Patterns	freq	area
$\overline{AB}$	2	4
$\overline{AEF}$	2	6
$B$	6	6
$BCDE$	4	16
$\overline{CDEF}$	2	8
$E$	6	6
$\vdots$	$\vdots$	$\vdots$



$|\mathcal{L}_{\mathcal{I}}| = 2^6$ , but only 4 skypatterns

$$\text{Sky}(\mathcal{L}_{\mathcal{I}}, \{\text{freq}, \text{area}\}) = \{BCDE, BCD, B, E\}$$

Problem	Skylines	Skypatterns
<b>Mining task</b>	a set of non dominated transactions	a set of non dominated patterns
<b>Size of the space search domain</b>	$ \mathcal{D} $	$ \mathcal{L} $
	a lot of works	very few works

usually:  $|\mathcal{D}| \ll |\mathcal{L}|$

$\mathcal{D}$	set of transactions
$\mathcal{L}$	set of patterns

A naive enumeration of all candidate patterns ( $\mathcal{L}_{\mathcal{I}}$ ) and then comparing them **is not feasible**...

## Two approaches:

- 1 take benefit from the **pattern condensed representation** according to the condensable measures of the given set of measures  $M$ 
  - **skylineability** to obtain  $M'$  ( $M' \subseteq M$ ) giving a more concise pattern condensed representation
  - the pattern condensed representation w.r.t.  $M'$  is a superset of the representative skypatterns w.r.t.  $M$  which is (much smaller) than  $\mathcal{L}_{\mathcal{I}}$ .
- 2 use of the **dominance programming framework**

**Dominance:** a pattern is optimal if it is not dominated by another.  
Skypatterns: dominance relation = Pareto dominance

## 1 Principle:

- starting from an initial pattern  $s_1$
- searching for a pattern  $s_2$  such that  $s_1$  is not preferred to  $s_2$
- searching for a pattern  $s_3$  such that  $s_1$  and  $s_2$  are not preferred to  $s_3$
- $\vdots$
- until there is no pattern satisfying the whole set of constraints

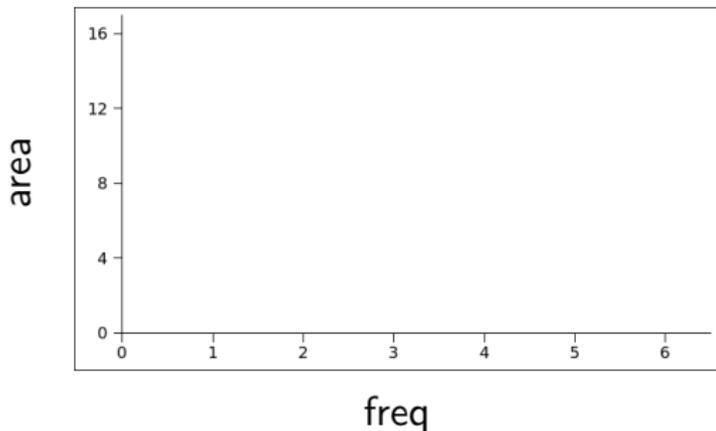
## 2 Solving:

- constraints are dynamically posted during the mining step

**Principle:** increasingly reduce the dominance area by processing **pairwise comparisons between patterns**. Methods using **Dynamic CSP** (Negrevergne et al. ICDM13, Ugarte et al. CPAIOR14).

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$		B			E	F
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



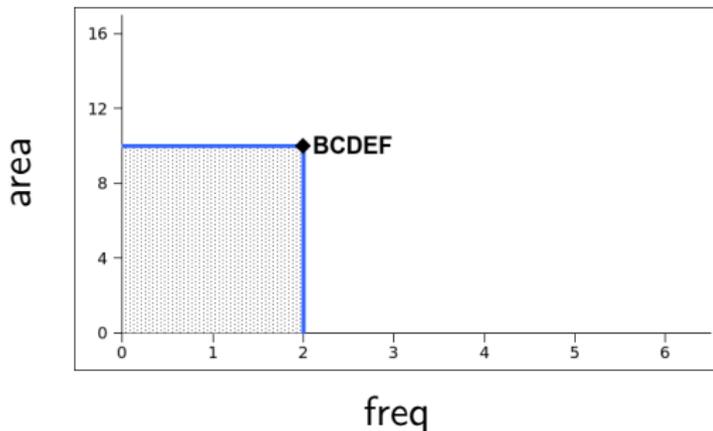
$$M = \{freq, area\}$$

$$q(X) \equiv closed_M(X)$$

Candidates =

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$		B			E	F
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



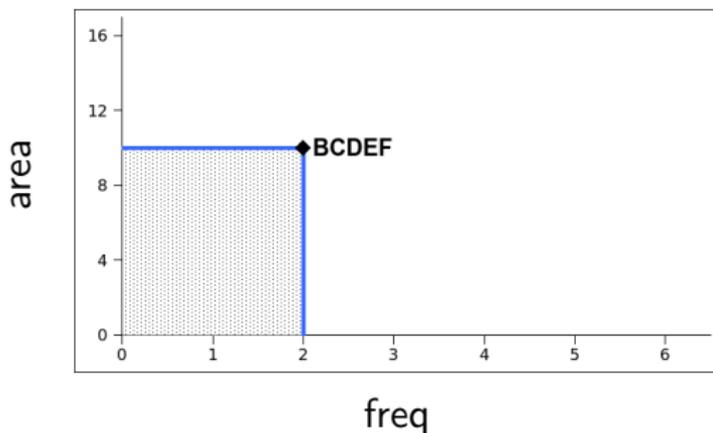
$$M = \{freq, area\}$$

$$q(X) \equiv closed_{M'}(X)$$

$$Candidates = \underbrace{\{BCDEF\}}_{s_1}$$

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$		B			E	F
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



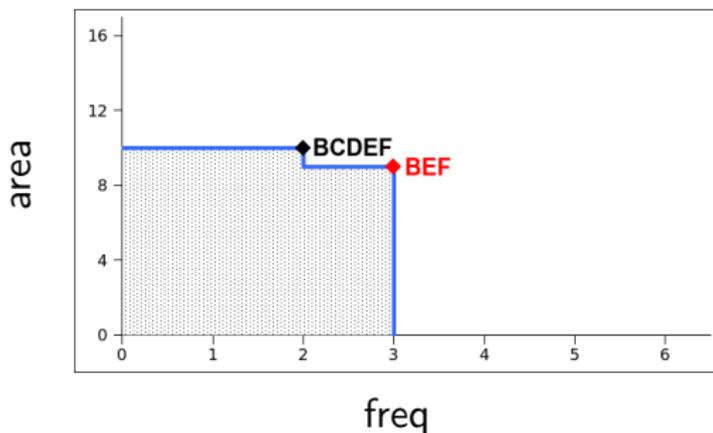
$$M = \{freq, area\}$$

$$q(X) \equiv closed_{M'}(X) \wedge \neg(s_1 \succ_M X)$$

$$Candidates = \underbrace{\{BCDEF\}}_{s_1}$$

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$		B			E	F
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



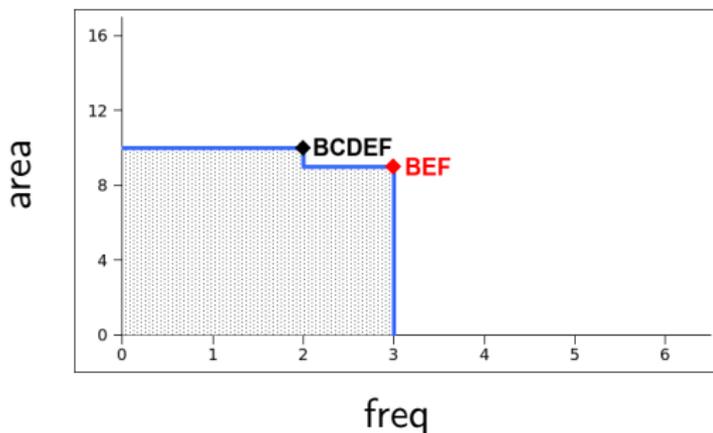
$$M = \{freq, area\}$$

$$q(X) \equiv \text{closed}_{M'}(X) \wedge \neg(s_1 \succ_M X)$$

$$\text{Candidates} = \underbrace{\{BCDEF\}}_{s_1}, \underbrace{\{BEF\}}_{s_2}$$

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$		B			E	F
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$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



$$M = \{freq, area\}$$

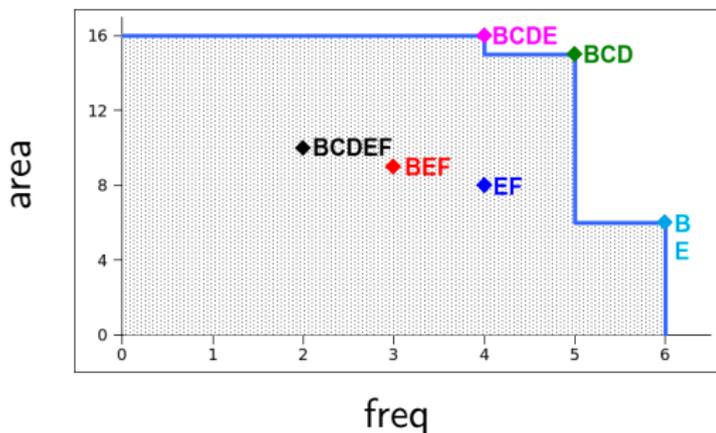
$$q(X) \equiv closed_{M'}(X) \wedge \neg(s_1 \succ_M X) \wedge \neg(s_2 \succ_M X)$$

$$Candidates = \underbrace{\{BCDEF\}}_{s_1}, \underbrace{\{BEF\}}_{s_2}$$

# Dominance programming example of the skypatterns

Trans.	Items					
$t_1$	B			E	F	
$t_2$	B	C	D			
$t_3$	A			E	F	
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F

$|\mathcal{L}_{\mathcal{I}}| = 2^6 = 64$  patterns  
4 skypatterns



$$M = \{freq, area\}$$

$$q(X) \equiv closed_{M'}(X) \wedge \neg(s_1 \succ_M X) \wedge \neg(s_2 \succ_M X) \wedge \neg(s_3 \succ_M X) \wedge \neg(s_4 \succ_M X) \wedge \neg(s_5 \succ_M X) \wedge \neg(s_6 \succ_M X) \wedge \neg(s_7 \succ_M X)$$

$$Candidates = \left\{ \underbrace{BCDEF}_{s_1}, \underbrace{BEF}_{s_2}, \underbrace{EF}_{s_3}, \underbrace{BCDE}_{s_4}, \underbrace{BCD}_{s_5}, \underbrace{B}_{s_6}, \underbrace{E}_{s_7} \right\}$$

$\underbrace{\hspace{15em}}_{Sky(\mathcal{L}_{\mathcal{I}}, M)}$

	dominance relation
maximal patterns	inclusion
closed patterns	inclusion at same frequency
top- $k$ patterns	order induced by the interestingness measure
skypatterns	Pareto dominance

maximal patterns  $\subseteq$  closed patterns

top- $k$  patterns  $\subseteq$  skypatterns

A **preference**  $\triangleright$  is a strict partial order relation on a set of patterns  $\mathbb{S}$ .  
 $x \triangleright y$  indicates that  $x$  is preferred to  $y$

(Ugarte et al. ICTAI15): a pattern  $x$  is **optimal** (OP) according to  $\triangleright$  iff  
 $\nexists y_1, \dots, y_p \in \mathbb{S}, \forall 1 \leq j \leq p, y_j \triangleright x$

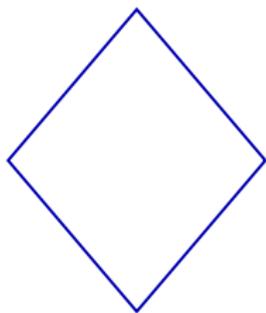
(a single  $y$  is enough for many data mining tasks)

**Characterisation of a set of OPs:** a set of patterns:

$$\left\{ x \in \mathbb{S} \mid \text{fundamental}(x) \wedge \nexists y_1, \dots, y_p \in \mathbb{S}, \forall 1 \leq j \leq p, y_j \triangleright x \right\}$$

**fundamental**( $x$ ):  $x$  must satisfy a **property** defined by the user  
for example: having a **minimal frequency**, being **closed**, ...

Trans.	Items					
$t_1$		B		E	F	
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



$$\mathbb{S} = \mathcal{L}_{\mathcal{I}}$$

(Mannila et al. DAMI97)

## Large tiles

$$c(x) \equiv \text{freq}(x) \times \text{size}(x) \geq \psi_{\text{area}}$$

Example:  $\text{freq}(BCD) \times \text{size}(BCD) = 5 \times 3 = 15$

## Frequent closed patterns

$$c(x) \equiv \begin{aligned} &\text{freq}(x) \geq \psi_{\text{freq}} \\ &\wedge \nexists y \in \mathbb{S} : y \supset x \\ &\wedge \text{freq}(y) = \text{freq}(x) \end{aligned}$$

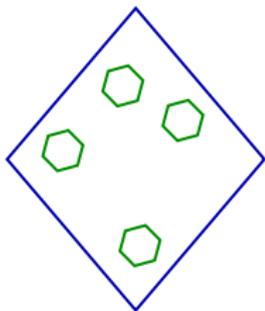
## Skypatterns

$$c(x) \equiv \begin{aligned} &\text{closed}_M(x) \\ &\wedge \nexists y \in \mathbb{S} : y \succ_M x \end{aligned}$$

## Frequent top-k patterns according to m

$$c(x) \equiv \begin{aligned} &\text{freq}(x) \geq \psi_{\text{freq}} \\ &\wedge \nexists y_1, \dots, y_k \in \mathbb{S} : \\ &\quad \bigwedge_{1 \leq j \leq k} m(y_j) > m(x) \end{aligned}$$

Trans.	Items					
$t_1$		B		E	F	
$t_2$		B	C	D		
$t_3$	A				E	F
$t_4$	A	B	C	D	E	
$t_5$		B	C	D	E	
$t_6$		B	C	D	E	F
$t_7$	A	B	C	D	E	F



$$\mathbb{S} = \mathcal{L}_{\mathcal{I}}$$

(Mannila et al. DAMI97)

Large tiles

$$c(x) \equiv \text{freq}(x) \times \text{size}(x) \geq \psi_{\text{area}}$$

Frequent closed patterns

$$c(x) \equiv \begin{aligned} &\text{freq}(x) \geq \psi_{\text{freq}} \\ &\wedge \nexists y \in \mathbb{S} : y \supset x \\ &\wedge \text{freq}(y) = \text{freq}(x) \end{aligned}$$

Skypatterns

$$c(x) \equiv \begin{aligned} &\text{closed}_M(x) \\ &\wedge \nexists y \in \mathbb{S} : y \succ_M x \end{aligned}$$

Frequent top-k patterns according to  $m$

$$c(x) \equiv \begin{aligned} &\text{freq}(x) \geq \psi_{\text{freq}} \\ &\wedge \nexists y_1, \dots, y_k \in \mathbb{S} : \\ &\quad \bigwedge_{1 \leq j \leq k} m(y_j) > m(x) \end{aligned}$$

**Patterns sets** (De Raedt and Zimmermann SDM07): sets of patterns satisfying a global viewpoint (instead of evaluating and selecting patterns based on their individual merits)

**Search space ( $\mathbb{S}$ ):** local patterns versus pattern sets

example:  $\mathcal{I} = \{A, B\}$

- all local patterns:  $\mathbb{S} = \mathcal{L}_{\mathcal{I}} = \{\emptyset, A, B, AB\}$

- all pattern sets:

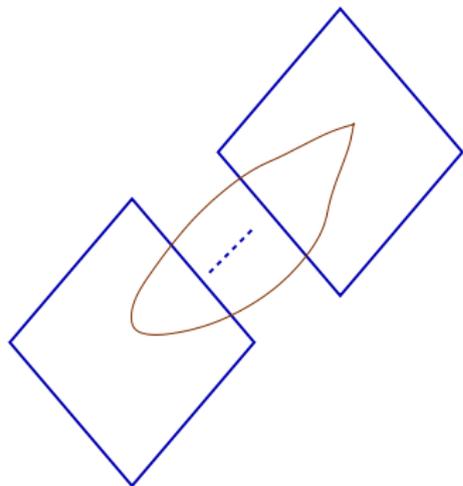
$$\mathbb{S} = 2^{\mathcal{L}_{\mathcal{I}}} = \{\emptyset, \{A\}, \{B\}, \{AB\}, \{A, B\}, \{A, AB\}, \{B, AB\}, \{A, B, AB\}\}$$

**Many data mining tasks:** classification (Liu et al. KDD98), clustering (Ester et al. KDD96), database tiling (Geerts et al. DS04), pattern summarization (Xin et al. KDD06), pattern teams (Knobbe and Ho PKDD06),...

**Many input (“preferences”) can be given by the user:**

coverage, overlapping between patterns, syntactical properties, measures, number of local patterns,...

# Pattern sets of length $k$ : examples



$\mathbb{S} \subset 2^{\mathcal{L}_{\mathcal{I}}}$   
(sets of length  $k$ )

*Conceptual clustering (without overlapping)*

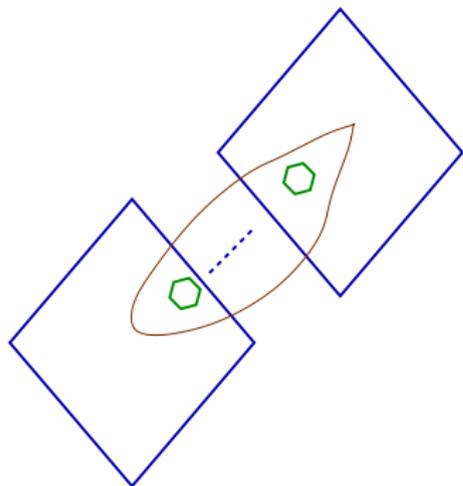
$$\text{clus}(x) \equiv \bigwedge_{i \in [1..k]} \text{closed}(x_i) \wedge \bigcup_{i \in [1..k]} \mathbb{T}(x_i) = \mathcal{T} \wedge \bigwedge_{i, j \in [1..k]} \mathbb{T}(x_i) \cap \mathbb{T}(x_j) = \emptyset$$

*Conceptual clustering with optimisation*

$$\text{c}(x) \equiv \text{clus}(x) \wedge \nexists y \in 2^{\mathcal{L}_{\mathcal{I}}}, \min_{j \in [1..k]} \{\text{freq}(y_j)\} > \min_{i \in [1..k]} \{\text{freq}(x_i)\}$$

*Pattern teams*

$$\text{c}(x) \equiv \text{size}(x) = k \wedge \nexists y \in 2^{\mathcal{L}_{\mathcal{I}}}, \Phi(y) > \Phi(x)$$

**(Optimal)** pattern sets of length  $k$ : examples

$\mathbb{S} \subset 2^{\mathcal{L}_{\mathcal{I}}}$   
(sets of length  $k$ )

*Conceptual clustering (without overlapping)*

$$\text{clus}(x) \equiv \bigwedge_{i \in [1..k]} \text{closed}(x_i) \wedge \bigcup_{i \in [1..k]} \mathbb{T}(x_i) = \mathcal{T} \wedge \bigwedge_{i, j \in [1..k]} \mathbb{T}(x_i) \cap \mathbb{T}(x_j) = \emptyset$$

*Conceptual clustering with optimisation*

$$\text{c}(x) \equiv \text{clus}(x) \wedge \nexists y \in 2^{\mathcal{L}_{\mathcal{I}}}, \min_{j \in [1..k]} \{\text{freq}(y_j)\} > \min_{i \in [1..k]} \{\text{freq}(x_i)\}$$

*Pattern teams*

$$\text{c}(x) \equiv \text{size}(x) = k \wedge \nexists y \in 2^{\mathcal{L}_{\mathcal{I}}}, \Phi(y) > \Phi(x)$$

**The idea:** the user as part of the process, he/she states expectations/beliefs, e.g.: number of items bought by customers, popularity of items, overall graph density (in dense subgraph mining)

➡ whatever contrasts with this = subjectively interesting

- producing a **set of patterns**: the background distribution is updated according to the patterns previously extracted
- **iterative approach**: at each step, the best pattern according the interestingness criterion is extracted (trade off between information content and descriptive complexity)

(Gallo et al. ECML/PKDD07, De Bie DAMI11, De Bie IDA13, van Leeuwen et al. MLJ16)

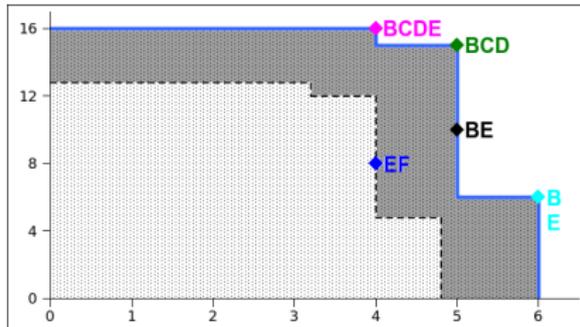
Recent work: interactive visual exploration (Puolamäki et al. ECML/PKDD16)

# Relax the dogma “must be optimal”: soft patterns

**Stringent aspect** of the classical constraint-based pattern mining framework: *what about a pattern which slightly violates a query?*

**example:** introducing **softness**  
in the skypattern mining:

➔ **soft**-skypatterns



put the user in the loop to determine the best patterns w.r.t. his/her preferences

Introducing softness is easy with Constraint Programming:

➔ **same process:** it is enough to **update the posted constraints**

## Examples: heuristic approaches

- *mining dense subgraphs* (Charalampos et al. KDD13)
- *pattern sets based on the Minimum Description Length principle:*
  - a small set of patterns that compress - KRIMP (Siebes et al. SDM06)
  - characterizing the differences and the norm between given components in the data - DIFFNORM (Budhathoki and Vreeken ECML/PKDD15)

Nice results based on the frequency. How handling other measures?

# Pattern mining as an optimization problem: concluding remarks

---

In the approaches indicated in this part:

- measures/preferences are **explicit** and must be given by the user... (but there is **no threshold** :-)
  - **diversity issue**: top- $k$  patterns are often very similar
  - **complete approaches** (optimal w.r.t the preferences):
    - ↳ **stop completeness** "Please, please stop making new algorithms for mining *all* patterns"
- Toon Calders (ECML/PKDD 2012, most influential paper award)

**A further step:** interactive pattern mining (including the instant data mining challenge), implicit preferences and learning preferences

Frequent pattern  
mining  
1995

Condensed  
representations  
2000

Pattern sets  
Top-k pattern mining  
2005

Optimal pattern mining  
Dominance programming  
2010

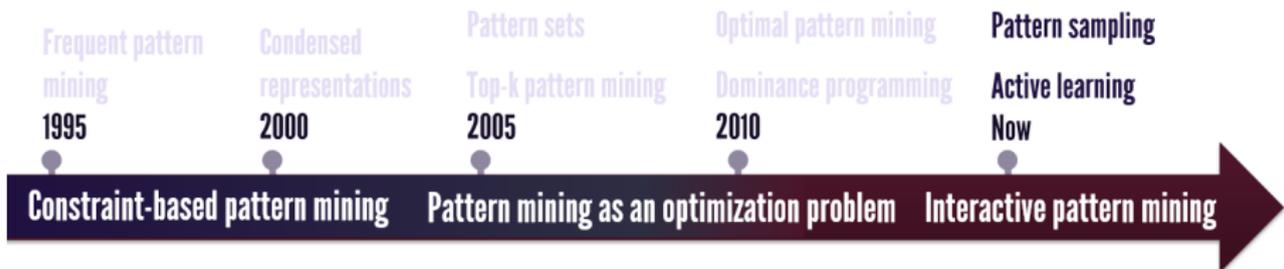
Pattern sampling  
Active learning  
Now

Constraint-based pattern mining

Pattern mining as an optimization problem

Interactive pattern mining

Interactive pattern mining



Idea: *"I don't know what I am looking for, but I would definitely know if I see it."*

⇒ preference acquisition

In this part:

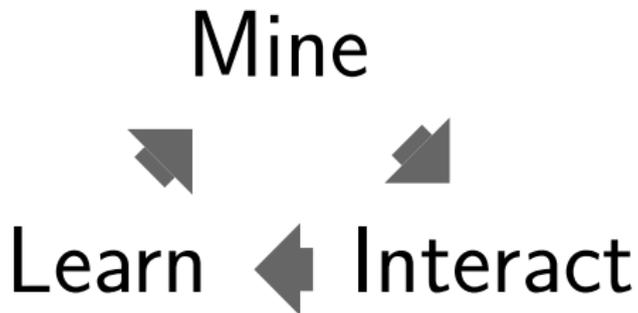
- Easier: no user-specified parameters (constraint, threshold or measure)!
- Better: learn user preferences from user feedback
- Faster: instant pattern discovery

## Advanced Information Retrieval-inspired techniques

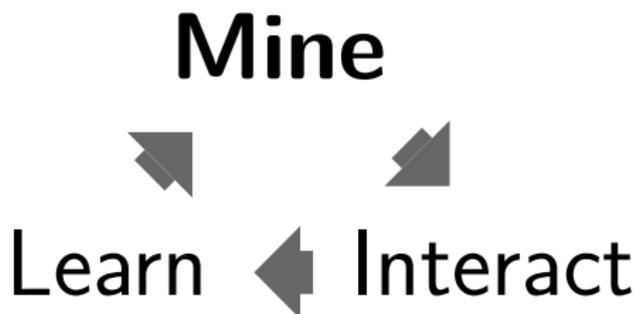
- Query by Example in information retrieval (QEIR) (Chia et al. SIGIR08)
- Active feedback with Information Retrieval (Shen et al. SIGIR05)
- SVM Rank (Joachims KDD02)
- ...

**Challenge:** pattern space  $\mathcal{L}$  is often much larger than the dataset  $\mathcal{D}$

 Interactive data exploration using pattern mining. (van Leeuwen 2014)



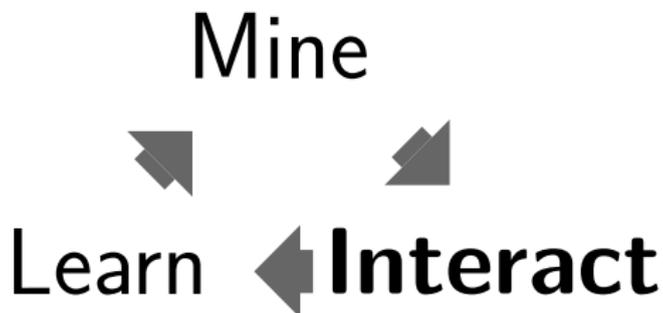
 Interactive data exploration using pattern mining. (van Leeuwen 2014)



## Mine

- Provide a sample of  $k$  patterns to the user (called the query  $\mathcal{Q}$ )

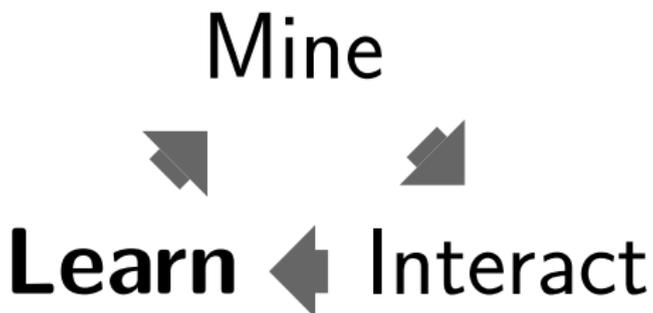
 Interactive data exploration using pattern mining. (van Leeuwen 2014)



## Interact

- Like/dislike or rank or rate the patterns

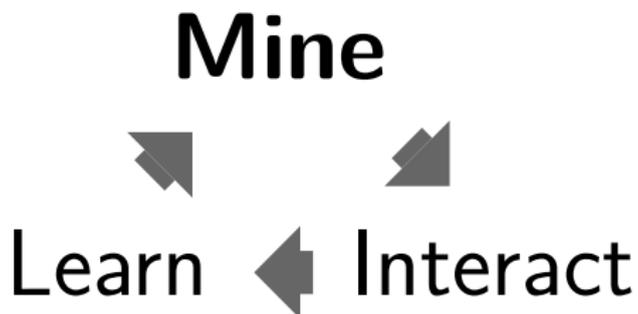
 Interactive data exploration using pattern mining. (van Leeuwen 2014)



## Learn

- Generalize user feedback for building a preference model

 Interactive data exploration using pattern mining. (van Leeuwen 2014)



## Mine (again!)

- Provide a sample of  $k$  patterns **benefiting from the preference model**

## Multiple mining algorithms

### Bonn Click Mining

A One-Click Mining Prototype by KDML Group, University of Bonn.

Test You are working on Test

Area Code	Area Name	CDU 2005	SPD 2005	FDP 2005	GREEN 2005	LEFT 2005	Electoral Participation 2005	CDU 2009	SPD 2009	FDP 2009	GREEN 2009	LEFT 2009	Population Density	Elderly population	Old Population	Middle-aged Population
9173	Bad Tölz-Wolfratshausen, Landkreis	55.9	17.9	11.3	8.8	2.4	79.9	46.7	12	17.1	11.2	4.6	106.2	20.6	27.4	26.1
9188	Starnberg, Landkreis	48.9	20.3	15.8	12.5	2.1	84.3	39.2	14.1	22.1	14.7	3.7	286.6	22.2	27.7	25.3
9175	Ebersberg, Landkreis	50.3	22.4	11.6	10.3	2.5	83.5	42.4	14.9	16.9	13.1	4.2	232.8	18.5	27	27.5
9172	Berchtesgadener Land, Landkreis	58.6	19.2	8.2	6.6	2.8	76.7	50.7	12.3	13.2	10.6	4.8	121.5	23.2	26.7	25.8
9177	Erding, Landkreis	55	20.3	9.4	7.4	2.9	79.5	45.5	12.4	14.7	12	4.8	145.1	15.6	27.3	28.9
9184	München, Landkreis	45.3	24.1	14.6	10.6	2.8	83.4	39.8	16.7	19.6	12.7	4.5	476.1	20.1	26.6	27.9
9176	Eichstätt, Landkreis	54.2	26.5	8.9	5.4	2.7	81.2	51.4	15.7	11.2	7.8	5.9	102.7	16.9	26.7	27.3
9182	Meerbach, Landkreis	54.8	19.2	12.8	7.6	2.4	80.3	48.1	12.2	17.6	10.2	3.9	116.6	21.9	27.3	26.1
9185	Neuburg-Schrobenhausen, Landkreis	57.6	22.1	7.9	4.8	3	77.5	52.6	13.2	13.5	7.2	5.7	122.4	18.1	27.5	26.9
9186	Pfaffenlofen a.d. Alm, Landkreis	53.1	23.7	9.3	6.4	3.2	78.3	49.3	13.7	14.1	9.1	5.6	151.8	17.1	28.1	27.6
9189	Traunstein, Landkreis	56.9	20.1	8.3	7.4	2.8	78.2	47.7	12.7	12.8	12.1	5.1	111.2	21.7	27.8	24.9
9173	Edenau, Landkreis	44.6	30.4	0.3	0.6	3	87.6	39.1	20.6	14.1	11.6	6	221.1	19.3	26.6	24.0

GREEN 2009=low;  
 FDP 2009=low;  
 Middle-aged Population=low;  
 ...  
 Frequency : 0.455437;  
 Dev. GREEN 2005: 0.145796;

Old Population=low;  
 Agricultural workforce=low;  
 No school degree=low;  
 ...  
 Frequency : 0.606796;  
 Dev. Construction workforce: 0.157288;

Children Population=high;  
 Finance workforce=low;  
 Population Density=low;  
 ...  
 Frequency : 0.570388;  
 Dev. Highest school degree: 0.044211;

GREEN 2009=low;  
 FDP 2009=low;  
 Middle-aged Population=low;  
 ...  
 Frequency : 0.487004;  
 Dev. GREEN 2005: 0.143077;

Public service workforce=low;  
 Middle-aged Population=low;  
 GREEN 2009=low;  
 ...

Old Population=low;  
 Agricultural workforce=low;  
 No school degree=low;  
 ...  
 Frequency : 0.604369;  
 Dev. Construction workforce

Public service workforce=low;  
 Middle-aged Population=low;  
 GREEN 2005=low;  
 ...  
 Frequency : 0.640777;  
 Dev. Young Population: 0



 One Click Mining - Interactive Local Pattern Discovery through Implicit Preference and Performance Learning. (Boley et al. IDEA13)

Platform that implements descriptive rule discovery algorithms suited for neuroscientists

The screenshot shows the h(odor) web interface. At the top, there is a search bar with the text "Search in results". Below it, the current pattern "Odors" is displayed, with a blue button labeled "Musk" and icons for heart and share. Underneath, the "Subgroups (all)" section shows a grid of 12 smaller pattern cards, each with a "musk" label and various associated terms and counts. On the right side, there are two control panels. The top one has a green "Next with 1 step" button, a red "Stop" button, and a "Skip 1 step" dropdown. The middle panel, titled "Descriptor distribution", shows a bar chart labeled "DISTRIBUTION OF THE DESCRIPTOR MW" with an x-axis ranging from 88 to 323. The bottom panel, titled "Settings", shows "Current level : 1", "Max result : 100", "Beam width: 50", and "Max redundancy : 2".

 h(odor): Interactive Discovery of Hypotheses on the Structure-Odor Relationship in Neuroscience. (Bosc et al. ECML/PKDD16 (demo))

- MINE
  - Instant discovery for facilitating the iterative process
  - Preference model integration for improving the pattern quality
  - Pattern diversity for completing the preference model
- INTERACT
  - Simplicity of user feedback (binary feedback  $>$  graded feedback)
  - Accuracy of user feedback (binary feedback  $<$  graded feedback)
- LEARN
  - Expressivity of the preference model
  - Ease of learning of the preference model

- MINE
    - *Instant discovery for facilitating the iterative process*
    - *Preference model integration for improving the pattern quality*
    - Pattern diversity for completing the preference model
  - INTERACT
    - Simplicity of user feedback (binary feedback  $>$  graded feedback)
    - Accuracy of user feedback (binary feedback  $<$  graded feedback)
  - LEARN
    - *Expressivity of the preference model*
    - Ease of learning of the preference model
- ⇒ Optimal mining problem (according to preference model)

- MINE
  - Instant discovery for facilitating the iterative process
  - Preference model integration for improving the pattern quality
  - *Pattern diversity for completing the preference model*
- INTERACT
  - *Simplicity of user feedback (binary feedback > graded feedback)*
  - *Accuracy of user feedback (binary feedback < graded feedback)*
- LEARN
  - Expressivity of the preference model
  - *Ease of learning of the preference model*

⇒ Active learning problem

*How user preferences are represented?*

## Problem

- Expressivity of the preference model
- Ease of learning of the preference model

*How user preferences are represented?*

## Problem

- Expressivity of the preference model
- Ease of learning of the preference model

## Weighted product model

- A weight on items  $\mathcal{I}$
- Score for a pattern  $X =$  product of weights of items in  $X$

- (Bhuiyan et al., CIKM12)
- |      |            |          |            |          |            |     |     |
|------|------------|----------|------------|----------|------------|-----|-----|
|      | $\omega_A$ |          | $\omega_B$ |          | $\omega_C$ |     |     |
| $AB$ | 4          | $\times$ | 1          |          |            | $=$ | 4   |
| $BC$ |            |          | 1          | $\times$ | 0.5        | $=$ | 0.5 |

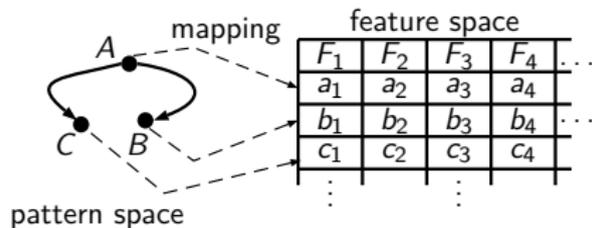
*How user preferences are represented?*

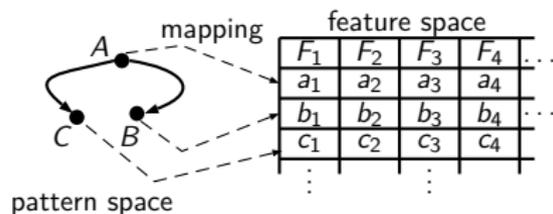
## Problem

- Expressivity of the preference model
- Ease of learning of the preference model

## Feature space model

- Partial order over the pattern language  $\mathcal{L}$
- Mapping between a pattern  $X$  and a set of features:





## Feature space

- = assumption about the user preferences
- the more, the better

Different feature spaces:

- Attributes of the mined dataset (Rueping ICML09)
- Expected and measured frequency (Xin et al. KDD06)
- Attributes, coverage, chi-squared, length and so on (Dzyuba et al. ICTAI13)

*How user feedback are represented?*

## Problem

- Simplicity of user feedback (binary feedback  $>$  graded feedback)
- Accuracy of user feedback (binary feedback  $<$  graded feedback)

*How user feedback are represented?*

## Problem

- Simplicity of user feedback (binary feedback  $>$  graded feedback)
- Accuracy of user feedback (binary feedback  $<$  graded feedback)

## Weighted product model

- Binary feedback (like/dislike) (Bhuiyan et al. CIKM12)

<u>pattern</u>	<u>feedback</u>
A	like
AB	like
BC	dislike

*How user feedback are represented?*

## Problem

- Simplicity of user feedback (binary feedback  $>$  graded feedback)
- Accuracy of user feedback (binary feedback  $<$  graded feedback)

## Feature space model

- Ordered feedback (ranking) (Xin et al. KDD06, Dzyuba et al. ICTAI13)

$$A \succ AB \succ BC$$

- Graded feedback (rate) (Rueping ICML09)

pattern	feedback
$A$	0.9
$AB$	0.6
$BC$	0.2

*How user feedback are generalized to a model?*

- **Weighted product model**

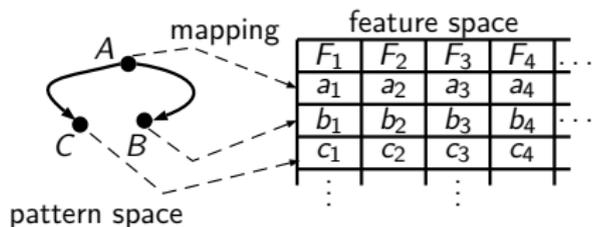
- Counting likes and dislikes for each item:  $\omega = \beta^{(\#\text{like} - \#\text{dislike})}$   
(Bhuiyan et al. ICML12)

pattern	feedback	A	B	C
A	like	1		
AB	like	1	1	
BC	dislike		-1	-1
		$2^{2-0} = 4$	$2^{1-1} = 1$	$2^{0-1} = 0.5$

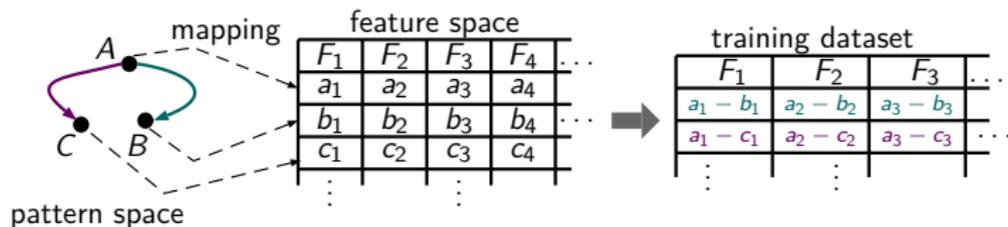
- **Feature space model**

- = learning to rank (Rueping ICML09, Xin et al. KDD06, Dzyuba et al. ICTAI13)

*How to learn a model from a ranking?*

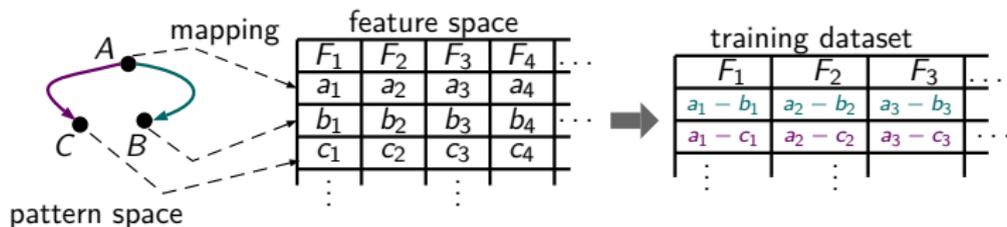


*How to learn a model from a ranking?*



- 1 Calculate the distances between feature vectors for each pair (training dataset)

*How to learn a model from a ranking?*



- 1 Calculate the distances between feature vectors for each pair (training dataset)
- 2 Minimize the loss function stemming from this training dataset

Algorithms: SVM Rank (Joachims KDD02), AdaRank (Xu et al. SIGIR07), ...

*How are selected the set of patterns (query  $Q$ )?*

## Problem

- Mining the most relevant patterns according to *Quality*
- Querying patterns that provide more information about preferences  
(NP-hard problem for pair-wise preferences (Ailon JMLR12))
- Heuristic criteria:
  - **Local diversity:** diverse patterns among the current query  $Q$
  - **Global diversity:** diverse patterns among the different queries  $Q_i$
  - **Density:** dense regions are more important

(Dzyuba et al. ICTAI13)

What is the interest of the pattern  $X$  for the current pattern query  $Q$ ?

- **Maximal Marginal Relevance:** querying diverse patterns in  $Q$

$$\alpha \text{Quality}(X) + (1 - \alpha) \min_{Y \in Q} \text{dist}(X, Y)$$

- **Global MMR:** taking into account previous queries

$$\alpha \text{Quality}(X) + (1 - \alpha) \min_{Y \in \bigcup_i Q_i} \text{dist}(X, Y)$$

- **Relevance, Diversity, and Density:** querying patterns from dense regions provides more information about preferences

$$\alpha \text{Quality}(X) + \beta \text{Density}(X) + (1 - \alpha - \beta) \min_{Y \in Q} \text{dist}(X, Y)$$

*What method is used to mine the pattern query  $Q$ ?*

## Problem

- Instant discovery for facilitating the iterative process
- Preference model integration for improving the pattern quality
- Pattern diversity for completing the preference model

*What method is used to mine the pattern query  $Q$ ?*

## Problem

- Instant discovery for facilitating the iterative process
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- Pattern diversity for completing the preference model

## Post-processing

- Re-rank the patterns with the updated quality (Rueping ICML09, Xin et al. KDD06)
- Clustering as heuristic for improving the local diversity (Xin et al. KDD06)

*What method is used to mine the pattern query  $Q$ ?*

## Problem

- Instant discovery for facilitating the iterative process
- Preference model integration for improving the pattern quality
- Pattern diversity for completing the preference model

## Optimal pattern mining (Dzyuba et al. ICTAI13)

- Beam search based on reweighing subgroup quality measures for finding the best patterns
- Previous active learning heuristics (and more)

*What method is used to mine the pattern query  $Q$ ?*

## Problem

- Instant discovery for facilitating the iterative process
- Preference model integration for improving the pattern quality
- Pattern diversity for completing the preference model

**Pattern sampling** (Bhuiyan et al. CIKM12)

- Randomly draw pattern with a distribution proportional to their updated quality
- Sampling as heuristic for diversity and density

## Methodology = simulate a user

- 1 Select a subset of data or pattern as **user interest**
- 2 Use a metric for simulating user feedback

### User interest:

- A set of items (Bhuiyan et al. CIKM12)
- A sample for modeling the user's prior knowledge (Xin et al. KDD06)
- A class (Rueping ICML09, Dzyuba et al. ICTAI13)

## Objective evaluation results

- Dozens of iterations for few dozens of examined patterns
- Important pattern features depends on the user interest
- Randomized selectors ensure high diversity

## Objective evaluation results

- Dozens of iterations for few dozens of examined patterns
- Important pattern features depends on the user interest
- Randomized selectors ensure high diversity

## Questions?

- How to select the right set of (hidden) features for modeling user preferences?
- How to subjectively evaluate interactive pattern mining?
  - ▣► qualitative benchmarks for pattern mining



Creedo – Scalable and Repeatable Extrinsic Evaluation for Pattern Discovery Systems by Online User Studies. (Boley et al. IDEA15)

## The need

*“the user should be allowed to pose and refine queries at any moment in time and the system should respond to these queries instantly”*

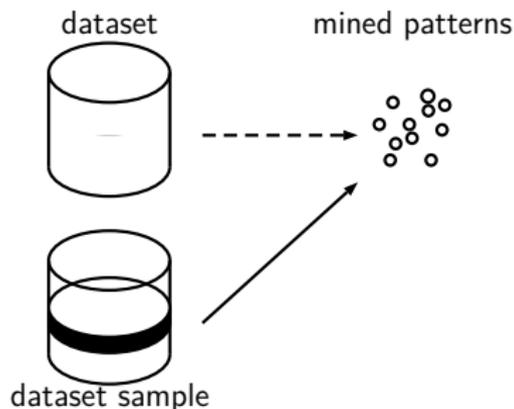
 Providing Concise Database Covers Instantly by Recursive Tile Sampling. (Moens et al. DS14)

▶ few seconds between the query and the answer

## Methods

- ~~Sound and complete pattern mining~~
- Beam search Subgroup Discovery methods
- **Pattern sampling**

## Dataset sampling



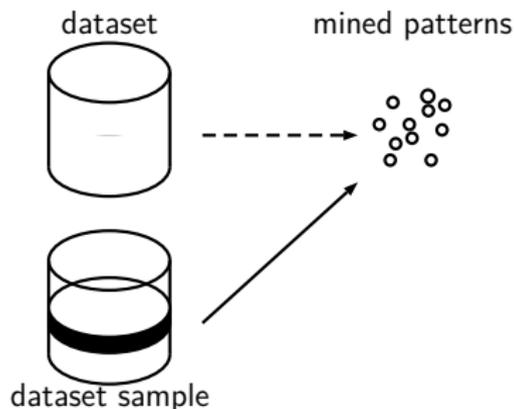
Finding all patterns from a transaction sample

⇒ input space sampling



Sampling large databases for association rules. (Toivonen et al. VLDB96)

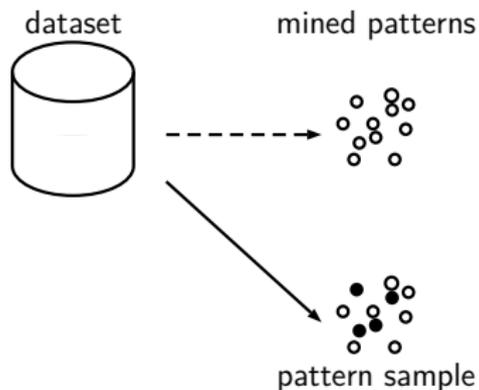
## Dataset sampling



Finding all patterns from a transaction sample

⇒ input space sampling

## Pattern sampling



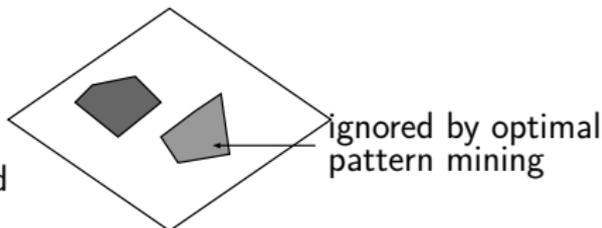
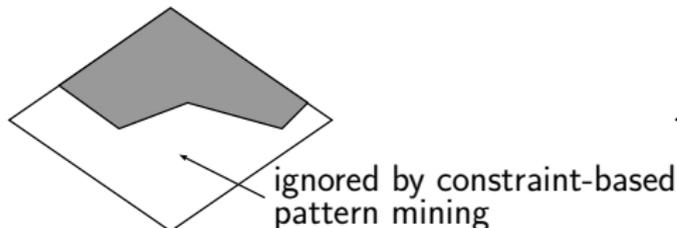
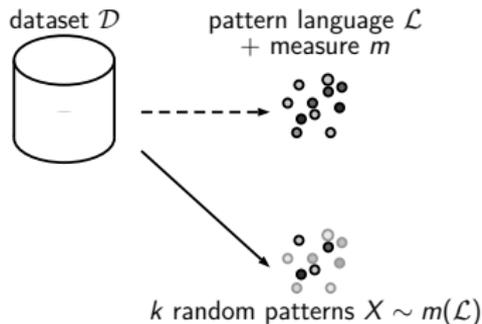
Finding a pattern sample from all transactions

⇒ output space sampling

-  Output Space Sampling for Graph Patterns. (Al Hasan et al. VLDB09)
-  Direct local pattern sampling by efficient two-step random procedures. (Boley et al. KDD11)
-  Interactive Pattern Mining on Hidden Data: A Sampling-based Solution. (Bhuiyan et al. CIKM12)
-  Linear space direct pattern sampling using coupling from the past. (Boley et al. KDD12)
-  Randomly sampling maximal itemsets. (Moens et Goethals IDEA13)
-  Instant Exceptional Model Mining Using Weighted Controlled Pattern Sampling. (Moens et al. IDA14)
-  Unsupervised Exceptional Attributed Sub-graph Mining in Urban Data (Bendimerad et al. ICDM16)

## Problem

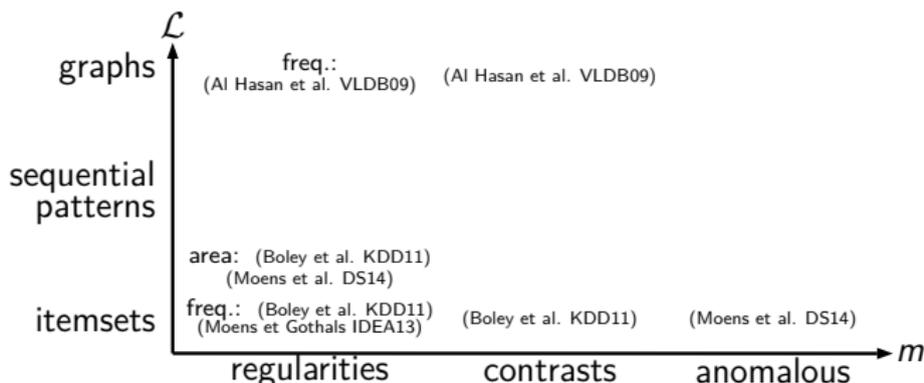
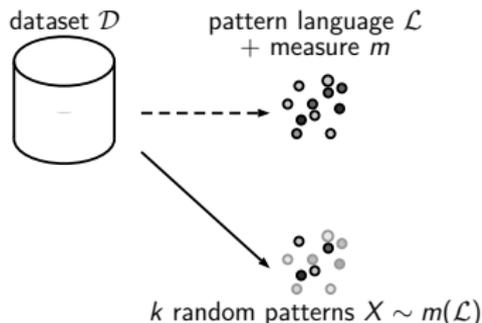
- **Inputs:** a pattern language  $\mathcal{L}$  + a measure  $m : \mathcal{L} \rightarrow \mathbb{R}$
- **Output:** a family of  $k$  realizations of the random set  $R \sim m(\mathcal{L})$



Pattern sampling addresses the full pattern language  $\mathcal{L}$   $\Rightarrow$  **diversity!**

## Problem

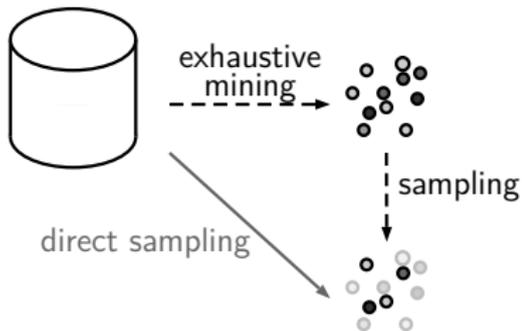
- **Inputs:** a pattern language  $\mathcal{L}$  + a measure  $m : \mathcal{L} \rightarrow \mathbb{R}$
- **Output:** a family of  $k$  realizations of the random set  $R \sim m(\mathcal{L})$



## Naive method

- 1 Mine all the patterns with their interestingness  $m$
- 2 Sample this set of patterns according to  $m$

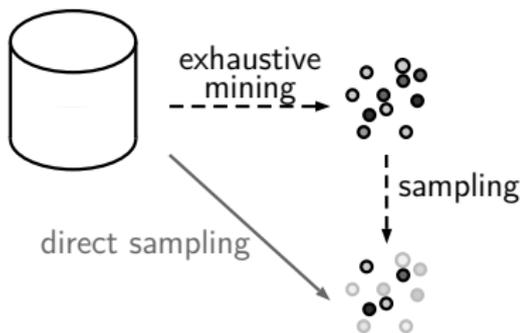
Time consuming / infeasible



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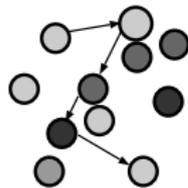


## Challenges

- Trade-off between pre-processing computation and processing time per pattern
- Quality of sampling

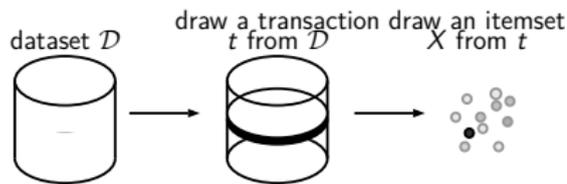
## 1. Stochastic techniques

- Metropolis-Hastings algorithm
- Coupling From The Past



## 2. Direct techniques

- Item/transaction sampling with rejection
- **Two-step random procedure**



# Two-step procedure: Toy example

 Direct local pattern sampling by efficient two-step random procedures.  
(Boley et al. KDD11)

Mine all frequent patterns

TId	Items		
$t_1$	A	B	C
$t_2$	A	B	
$t_3$		B	C
$t_4$			C

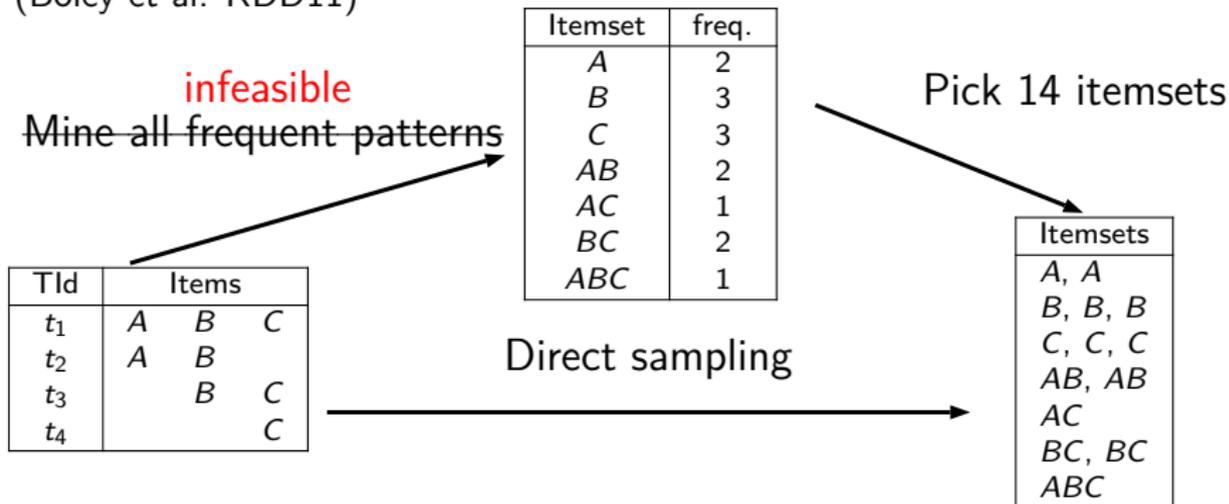
Itemset	freq.
A	2
B	3
C	3
AB	2
AC	1
BC	2
ABC	1

Pick 14 itemsets

Itemsets
A, A
B, B, B
C, C, C
AB, AB
AC
BC, BC
ABC

# Two-step procedure: Toy example

 Direct local pattern sampling by efficient two-step random procedures.  
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# Two-step procedure: Toy example

 Direct local pattern sampling by efficient two-step random procedures.  
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~~Mine all frequent patterns~~ **infeasible**

TId	Items
$t_1$	A B C
$t_2$	A B
$t_3$	B C
$t_4$	C

Itemset	freq.
A	2
B	3
C	3
AB	2
AC	1
BC	2
ABC	1

Pick 14 itemsets

Itemsets
A, A
B, B, B
C, C, C
AB, AB
AC
BC, BC
ABC

TId	Itemsets
$t_1$	A, B, C, AB, AC, BC, ABC
$t_2$	A, B, AB
$t_3$	B, C, BC
$t_4$	C

Rearrange itemsets

# Two-step procedure: Toy example

 Direct local pattern sampling by efficient two-step random procedures.  
(Boley et al. KDD11)

~~Mine all frequent patterns~~  
**infeasible**

TId	Items	weight $\omega$
$t_1$	A B C	$2^3 - 1 = 7$
$t_2$	A B	$2^2 - 1 = 3$
$t_3$	B C	$2^2 - 1 = 3$
$t_4$	C	$2^1 - 1 = 1$

Itemset	freq.
A	2
B	3
C	3
AB	2
AC	1
BC	2
ABC	1

Pick 14 itemsets

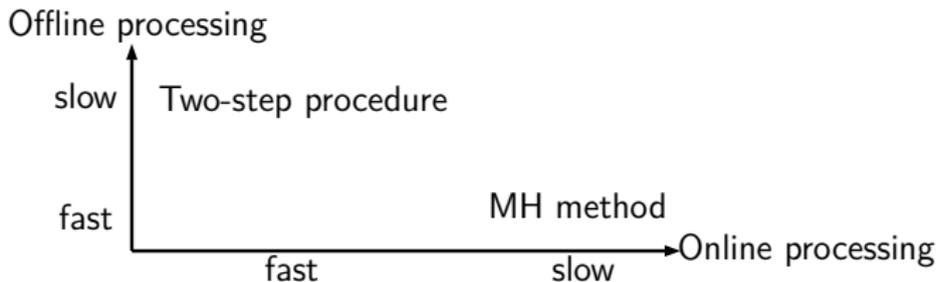
Itemsets
A, A
B, B, B
C, C, C
AB, AB
AC
BC, BC
ABC

1. Pick a transaction proportionally to  $\omega$

TId	Itemsets
$t_1$	A, B, C, AB, AC, BC, ABC
$t_2$	A, B, AB
$t_3$	B, C, BC
$t_4$	C

2. Pick an itemset uniformly

# Two-step procedure: Comparison

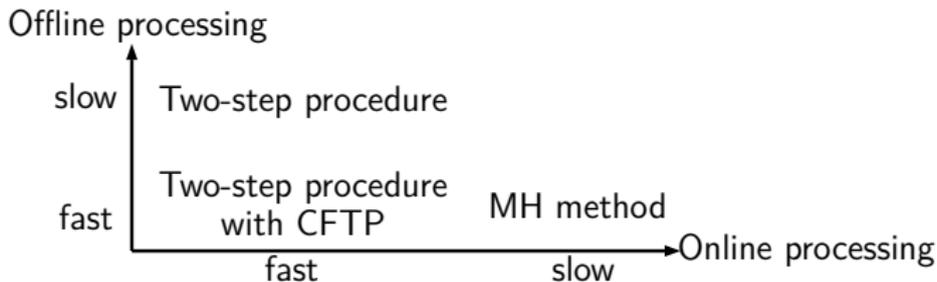


Complexity depends on the measure  $m$ :

Measure $m(X)$	Preprocessing	$k$ realizations
$\text{supp}(X, \mathcal{D})$	$O( \mathcal{I}  \times  \mathcal{D} )$	$O(k( \mathcal{I}  + \ln  \mathcal{D} ))$
$\text{supp}(X, \mathcal{D}) \times  X $	$O( \mathcal{I}  \times  \mathcal{D} )$	$O(k( \mathcal{I}  + \ln  \mathcal{D} ))$
$\text{supp}_+(X, \mathcal{D}) \times ( \mathcal{D}_-  - \text{supp}_-(X, \mathcal{D}))$	$O( \mathcal{I} ^2 \times  \mathcal{D} ^2)$	$O(k( \mathcal{I}  + \ln^2  \mathcal{D} ))$
$\text{supp}(X, \mathcal{D})^2$	$O( \mathcal{I} ^2 \times  \mathcal{D} ^2)$	$O(k( \mathcal{I}  + \ln^2  \mathcal{D} ))$

Preprocessing time may be prohibitive

# Two-step procedure: Comparison



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Measure $m(X)$	Preprocessing	$k$ realizations
$\text{supp}(X, \mathcal{D})$	$O( \mathcal{I}  \times  \mathcal{D} )$	$O(k( \mathcal{I}  + \ln  \mathcal{D} ))$
$\text{supp}(X, \mathcal{D}) \times  X $	$O( \mathcal{I}  \times  \mathcal{D} )$	$O(k( \mathcal{I}  + \ln  \mathcal{D} ))$
$\text{supp}_+(X, \mathcal{D}) \times ( \mathcal{D}_-  - \text{supp}_-(X, \mathcal{D}))$	$O( \mathcal{I} ^2 \times  \mathcal{D} ^2)$	$O(k( \mathcal{I}  + \ln^2  \mathcal{D} ))$
$\text{supp}(X, \mathcal{D})^2$	$O( \mathcal{I} ^2 \times  \mathcal{D} ^2)$	$O(k( \mathcal{I}  + \ln^2  \mathcal{D} ))$

Preprocessing time may be prohibitive  $\Rightarrow$  hybrid strategy with stochastic process for the first step:



Linear space direct pattern sampling using coupling from the past.

(Boley et al. KDD12)

## Summary

### Pros

- Compact collection of patterns
- Threshold free
- Diversity
- Very fast

### Cons

- Patterns far from optimality
- Not suitable for all interestingness measures

## Summary

### Pros

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## Interactive pattern sampling



Interactive Pattern Mining on Hidden Data: A Sampling-based Solution. (Bhuiyan et al. CIKM12)

➡ how to integrate more sophisticated user preference models?

## Pattern-based models with iterative pattern sampling

-  ORIGAMI: Mining Representative Orthogonal Graph Patterns. (Al Hasan et al. ICDM07)
-  Randomly sampling maximal itemsets. (Moens et Goethals IDEA13)
-  Providing Concise Database Covers Instantly by Recursive Tile Sampling. (Moens et al. DS14)

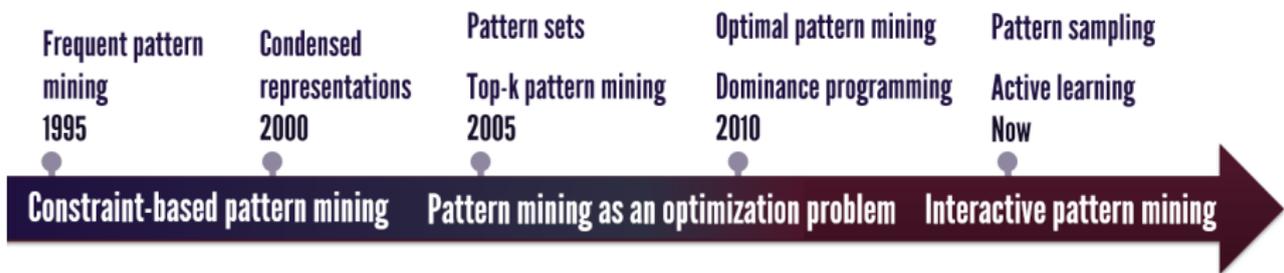
⇒ how to sample a set of patterns instead of individual patterns?

# Interactive pattern mining: concluding remarks

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- Preferences are not explicitly given by the user. . .  
. . . but, representation of user preferences should be anticipated in upstream.
  
- Instant discovery enables a tight coupling between user and system. . .  
. . . but, most advanced models are not suitable.

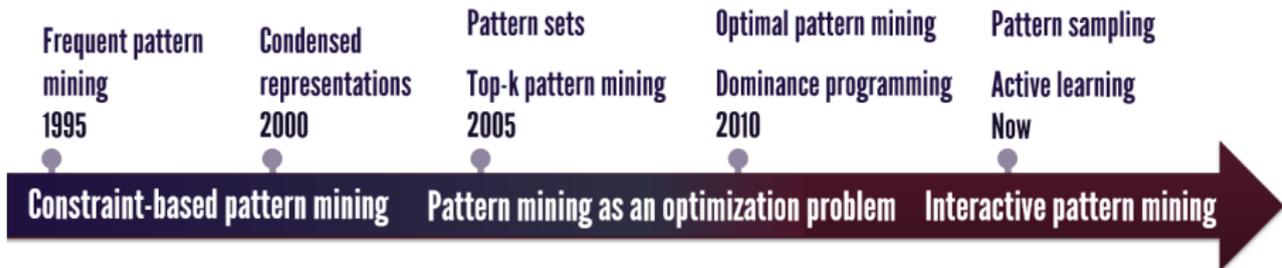
## Concluding remarks



User preferences are more and more prominent. . .

from simple preference models to complex ones

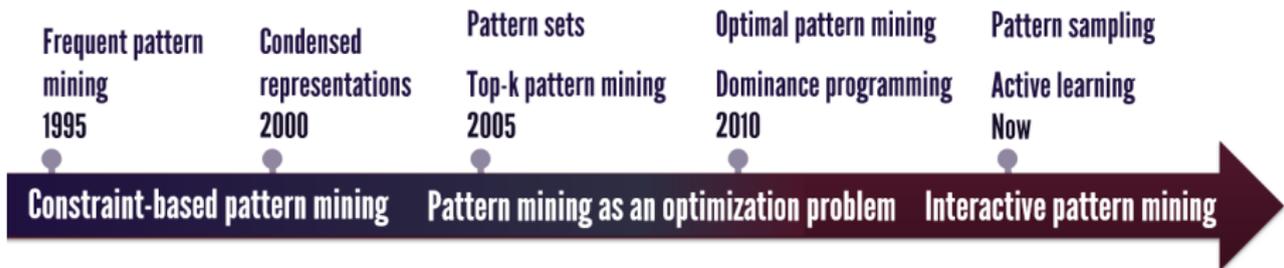
- from frequency to anti-monotone constraints and more complex ones
- from 1 criterion (top-k) to multi-criteria (skyline)
- from weighted product model to feature space model



User preferences are more and more prominent. . .

## from preference elicitation to preference acquisition

- user-defined constraint
- no threshold with optimal pattern mining
- no user-specified interestingness



User preferences are more and more prominent in the community. . .

from data-centric methods:

- 2003-2004: Frequent Itemset Mining Implementations
- 2002-2007: Knowledge Discovery in Inductive Databases

to user-centric methods:

- 2010-2014: Useful Patterns
- 2015-2016: Interactive Data Exploration and Analytics

## How to improve pattern mining for a user benefiting from other users?

- on the same dataset
- on a different dataset

## Information Retrieval inspired techniques?

- collaborative filtering
  - 📄 Combining collaborative filtering and sequential pattern mining for recommendation in e-learning environment. (Li et al. ICWL11)
- crowdsourcing

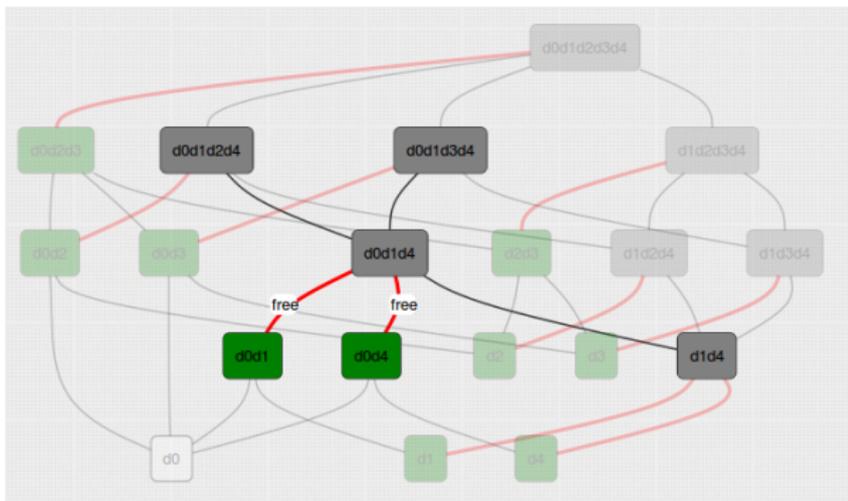
- The user has to choose its pattern domain of interest.
- What about (interactive) multi-pattern domain exploration?
  - Some knowledge nuggets can be depicted with simple pattern domain (e.g., itemset) while others require more sophisticated pattern domain (e.g., sequence, graph, dynamic graphs, etc.).
  - Examples in Olfaction:
    - Odorant molecules.
    - unpleasant odors in presence of Sulfur atom in chemicals  $\Rightarrow$  itemset is enough.
    - Some chemicals have the same 2-d graph representation and totally different odor qualities (e.g., isomers)  $\Rightarrow$  need to consider 3-d graph pattern domain.
  - How to fix the good level of description?
- Toward pattern sets involving several pattern domains.

# Multi optimization ... and user navigation

Concise representation of the skypattern cube:

→ equivalence classes on measures highlight the role of measures

- multi optimization:  
interest in sets of pattern sets  
(e.g., skypattern cube)  
( $2^{2^{\mathcal{I}}}$ )
- user navigation  
through the set of patterns
- recommendation



Iris data set: d0 = freq, d1 = max(val), d2 = mean(val),  
d3 = area, d4 = gr 1

<https://sdmc.greyc.fr/skypattern/> (P. Holat)

- **cross-fertilization between data mining and constraint programming** (De Raedt et al. KDD08): designing **generic** and **declarative** approaches
  - ➔ make easier the exploratory data mining process
    - avoiding writing solutions from scratch
    - easier to model new problems
- **pattern sets** are integrated in a natural way (Khiari et al. CP10, Guns et al. TKDE13)
- from itemsets to **other pattern languages**: sequences (Aoga et al. ECML/PKDD16, Kemmar et al. CPAIOR16)
- CP but also ILP (Babaki et al., CPAIOR14), SAT (Jabbour et al., CIKM13), ASP (Gebser et al. IJCAI16)
- several workshops: DPM 11 (Declarative Pattern Mining), CoCoMile 12 13 (Combining CONstraint solving with MIning and LEarning), Languages for DM/ML 13. Dagstuhl seminar 11 14.

French science academy

“The renewed success of AI”



35<sup>ans</sup>  
Académie des sciences

Le public scientifique

Conférence-débat  
Intelligence Artificielle:  
le renouveau

Découvrir, apprendre, reconnaître, juger, décider : ces tâches perceptives et cognitives que l'on associe à l'intelligence humaine deviennent chaque jour plus accessibles à l'automatisation. Grâce aux progrès considérables de la microélectronique, à la puissance de calcul qu'elle permet et à l'accès à des quantités gigantesques de données, l'Intelligence Artificielle (IA) vit aujourd'hui un renouveau qui s'appuie sur presque toutes les sciences et touche de plus en plus à notre vie quotidienne. L'objet de cette conférence-débat est d'en présenter quelques facettes actuelles remarquables, entre sciences de l'information et informatique neuro-inspirée.

4  
octobre  
2016

- **open issues:**

- how go further to integrate **preferences**, define **constraints**, **model a problem**?

Directions: defining languages?, learning constraints

- how to **visualize results** and **interact** with the end user?
- scaling
- ...

- **but also:**

- *the opposite direction is also a topic of interest*: how can constraint programming benefit from data mining techniques?
- results in ILP/SAT used in certain *probabilistic models* (Chang et al. AAAI08, Cussens UAI08)
- many other directions associated to the AI field: *integrating background knowledge, knowledge representation,...*

Likely a promising avenue:

many papers at ECAI16, IJCAI16 and [ECML/PKDD16!](#)

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