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Fitting a 3D Particle System Model to a Non-dense Data Set in Medical Applications

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Abstract. In the scope of a radiotherapy application, we have previously developed a methodology to reconstruct the 3D shape of deformable organs from their CT scan sections using particle systems. In this paper, we propose a new technique to track changes in the object's shape from some partial information provided by an ultrasound sensor. The aim of this method is to quickly obtain the shape alteration while preserving organ's volume which can be a strong requirement in medical applications. Our approach is based on the deformation of the initial model by applying physical forces to its surface in order to fit the new data set. The amplitude of this force is calculated to preserve the cohesion of the organ's model. The advantages of our method include:

- updating is faster than initial modelling,
- undersampled data is sufficient for updating.

Key words: deformable model, particle systems, physically based interpolation and modelling, Lennard Jones forces

MSC 2000: 68U05

1. Introduction

Computer graphics are increasingly used in medical applications. For example, computer aided surgery is becoming an important emerging topic in the medical world. Combined with haptical systems, deformable organs 3D models are used to simulate surgical operations. This helps the training of young surgeons and allows the experimented ones to perform less invasive surgeries [2, 3, 4].

In cancer's conformal radiotherapy, the models can also be used to accomplish a better treatment planning, dosimetric calculation and tumor targeting. However, the radiotherapy of cancer necessitates several treatment sessions and it can last several weeks. During this

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time organ's shape will evolve. Thus, the geometric models describing these shapes should be rebuilt according to the organs' change. A good solution should be to reconstruct the organ's model whenever the physicians think that the organ has undergone significant shape change between two successive treatments.

The reconstruction process needs a new re-sampling of organs' surfaces by means on invasive data capture techniques such as CT scans. Ethically and medically this can hardly be accepted. Therefore, non-invasive methods such as ultrasound (US) imaging techniques should be chosen. Unfortunately, the US images are too sensitive to the change of mass density and cause lots of diffraction and noise. They are generally less precise compared to CT scan images. The segmentation of US images yield generally to unstructured and under sampled data points. Consequently, they cannot be directly used for the reconstruction.

In this paper we present our approach: the organ's initial model is first reconstructed from the CT scan data. For this we use particle systems that permit to control volume and elasticity of the organ [1]. Next, the initial model is modified by applying forces on its surface to fit the data-sets extracted from US images.

1.1. Related works

There are some research works dealing with the warping of an initial geometric model to fit some new data points. The problem has been mostly studied from a geometrical point of view and the physical aspects of constant volume organ's constrained deformation has received less interest.

Turk [9] develops an implicit surface interpolation with a linear solution. Zindy [10] proposes a warping interpolation founded on the super quadrics models and FFD (Free Form Deformation). Bardinet [2] uses super quadrics to follow cardiac motion.

The geometric updating methods give good results but either need a sufficient panel of data, or need human interaction. Furthermore, in the Bardinet case, the chosen model seems to be adapted only for objects homeomorphic to spheres.

In our knowledge, there is few research works on the update of physically based models. These works, basically using mass/spring models, are not well adapted to volume control constraint. They are usually used to apply forces and observe dynamics and haptic behavior. The simulation of the behavior of these objects are obtained in "interactive" time, thanks to approximation techniques such as developed by James and Pai in their haptical finite element model [6]. Other works like Debunne's [4], propose hybrid models to accelerate calculation.

1.2. Our contribution

We present a method to modify particle systems model's shape from few data sets scanned on or beneath the surface of the object to be modeled. This is a physical interpolation that preserves the volume and which complexity depends on the number of initial data and the number of particles composing the model. Moreover, our method is automatic and does not need any interactions.

In Section 2 we present our previous results on which our method is based. In Section 3 our physically based updating model is introduced. Finally, in Section 4 we discuss the obtained results and give our conclusions about our method.

2. Background

In this section, we first present the particle systems which is the basis of our model. Secondly, we analyse the cohesion force that we use later for our purpose.

2.1. Particle Systems

Particle systems consist in a set of solid spheres whose movement follows physical laws. Particle systems were originally used to represent explosion, fire, cloud and any kind of objects that do not have any clear apparent bound. For deformable objects modelling, this simple model has been improved by the introduction of internal forces between particles to preserve the cohesion of the object. TONNESEN [8] applied the *Lennard Jones (L.J.) potential energy function*. As the corresponding forces to the L.J. potential energies are conservative, the system may oscillate. Thus damping forces are introduced to prevent system's instability. AMRANI [1] proposes to integrate a skeleton inside the particle system to prevent spherical clustering of the system and to maintain the global shape.

2.2. Lennard Jones' cohesion force

The Lennard Jones' force derives from molecular dynamics and is deduced from Lennard Jones' potential (ϕ):

$$LJ\,(r_{ij}) = -\nabla(\phi). \tag{1}$$

In (1), ∇ represents the gradient of the function ϕ. The amplitude of the interaction forces between two particles depends on their relative distance r_{ij} (eq. (2)).

$$LJ\,(r_{ij}) = \frac{-mnE_0}{(n-m)\,r_0}\left(\left(\frac{r_0}{r_{ij}}\right)^{(n+1)} - \left(\frac{r_0}{r_{ij}}\right)^{(m+1)}\right). \tag{2}$$

The parameters of this equation are :

- r_{ij}, the distance between two particles i and j,
- r_0, the distance of equilibrium between 2 particles,
- E_0, the cohesion energy,
- m, the attractive parameter between particles,
- n, the repulsive parameter between particles.

We show the associated curve in Fig. 1.

On this curve we can easily distinguish the long range attractive part ($r_{ij} > r_0$) and the short range repulsive part ($r_{ij} < r_0$).

From here, we can define the maximal force that the system cohesion can support without introducing instability (eq. (3)).

$$LJ'(r_{min}) = 0 \iff r_{min} = r_0 \sqrt[m-n]{\frac{m+1}{n+1}}\,. \tag{3}$$

The maximal forces between two particles that can be applied to ensure the system cohesion is then expressed in (4).

$$F_{max} = \frac{-E_0 mn}{(m-n)r_0}\left(\left(\sqrt[m-n]{\frac{n+1}{m+1}}\right)^{n+1} - \left(\sqrt[m-n]{\frac{n+1}{m+1}}\right)^{m+1}\right) \tag{4}$$

This maximal force will be used to determinate the update force in 3.3.

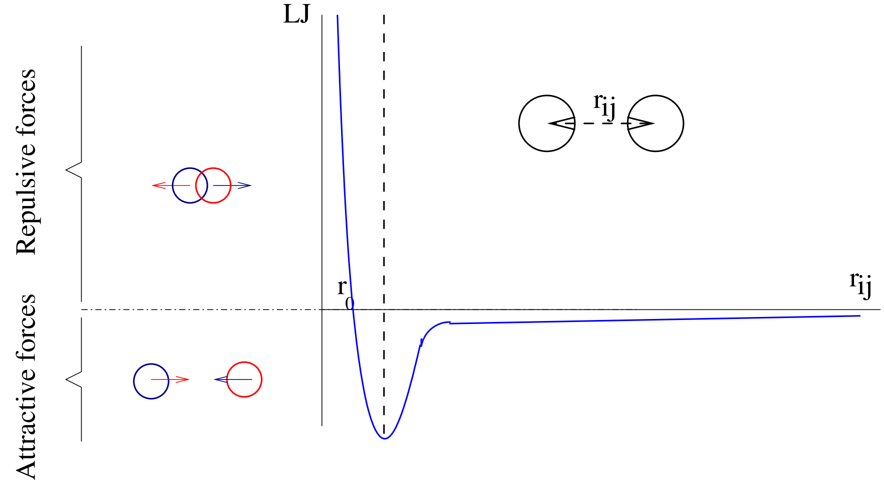


Figure 1: Lennard Jones' force according to the distance between particles

3. Physically based modification of a particle system

A physical system can be easily deformed by applying external constraints. Thus, we apply forces to our model to fit a sub-sampled dataset that we call attractive points.

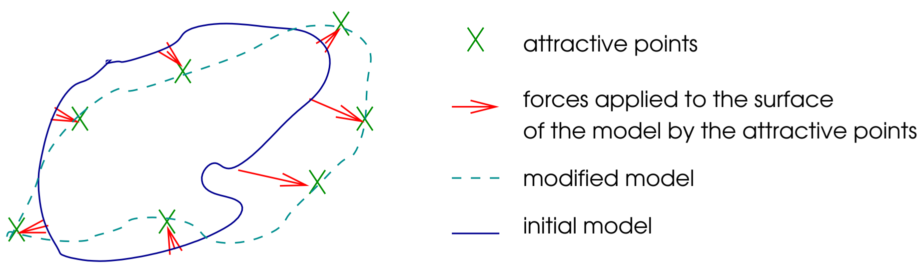


Figure 2: Intuitive modification method to fit a sub-sampled dataset

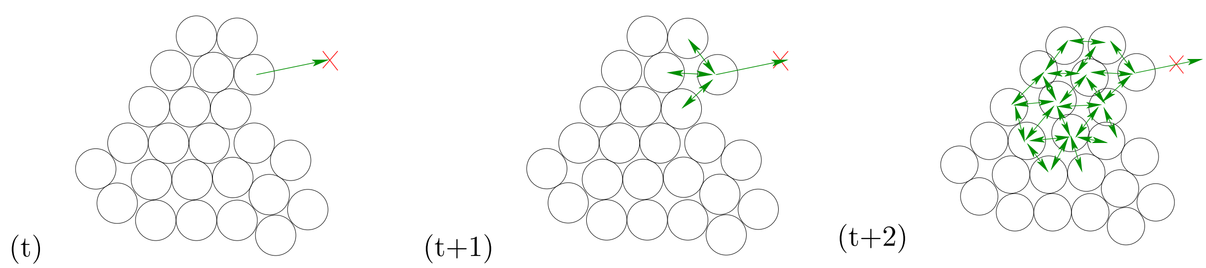


Figure 3: Cohesion force permits the system's deformation under external forces

The method that seems to be the most natural is to apply forces to the surface ("skin") of our model as shown in Fig. 2. Indeed, we locally interact with the model and let the internal cohesion forces modify the model shape to fit the attractive points (Fig. 3).
For this, we must determine the "skin particles" because there is no mesh fixed on the particle system's surface. Once the particles lying on the surface have been detected, we should define the force that should be applied to update the model as efficiently as possible.

3.1. Computation of the particles of the skin

We use a voxel based method to find the skin particles. In the following, we define the neighborhood of a voxel V_i as the set of all the voxels V_j with $j \neq i$, that are in contact with V_i. This means that 8 in 2D and 26 in 3D is the maximum number of neighboring voxels, that is called *full neighborhood*. Figure 4 illustrates the 2D case (3D case is straightforward):

1. We voxelize the bounding box of the particle system.
2. We preserve only the intersecting voxels.
3. We delete all the voxels that have a full neighborhood.
4. The skin particle are the ones intersecting the remaining voxels.

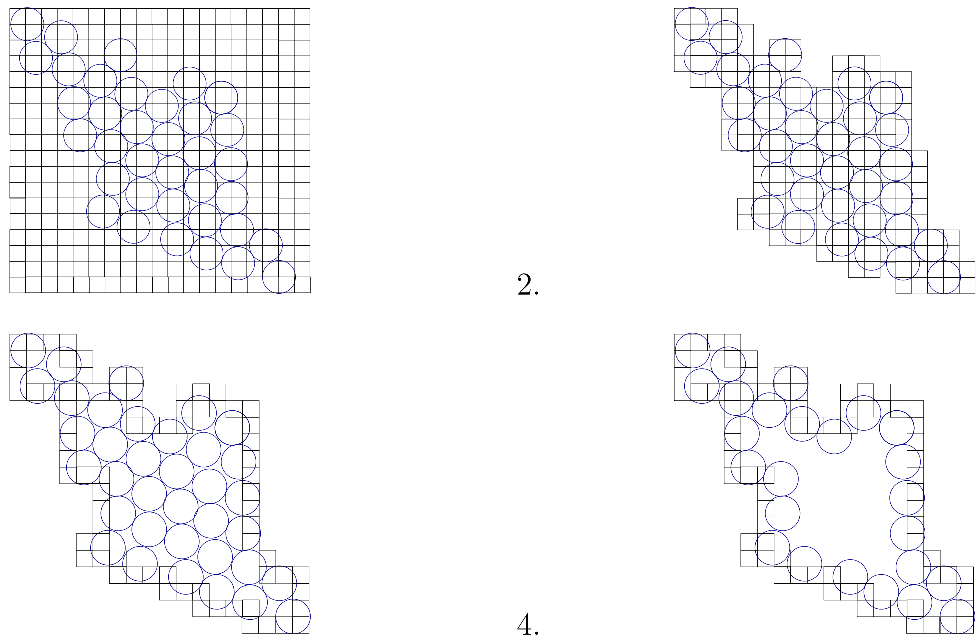

1. 2. 3. 4.

Figure 4: Voxel based method to find the skin particles

3.2. Constraints definition

While computing the skin, we determine the force range of the attractive points to the skin particles. We apply some intuitive constraints (see Figs. 6 and 5):

1. the furthest attractive point from the object must induce the strongest force and attract more particles because of the greatest deformation it must induce
2. contrary, among the particles attracted by the attractive point, the nearest particle, called *epicenter particle*, receives the maximum influence.

In Fig. 5 the first constraint is described: the number of particles of the skin influenced by each attractive point increases according to the distance of the attractive point to the object. The attractive point 1 attracts more particles of the skin than the attractive point 3 because 1 is further to the object than 3, but it attracts less than 2 because the attractive point 2 is further than 1.

In Fig. 6 the second constraint is described, showing that the most attracted particle of the skin is the nearest to the attractive point. Three cases are possible:

1. the particles of the skin are pulled through the attractive point outside of the system,

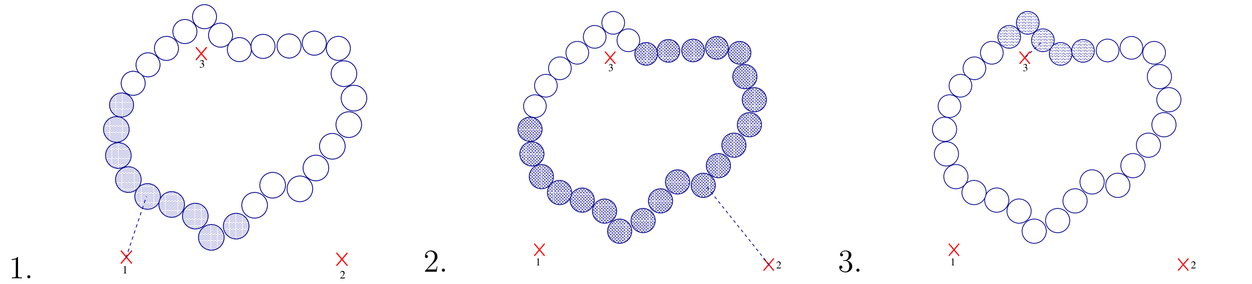


Figure 5: The attractive point influence depends on its distance to the system (the attractive point 1 has a greater influence than the attractive points 2 or 3 because of its greater distance).

2. the attractive point is inside a particle of the skin, no force is applied,
3. the attractive point is inside the particle system and the particles of the skin are repelled inside the system.

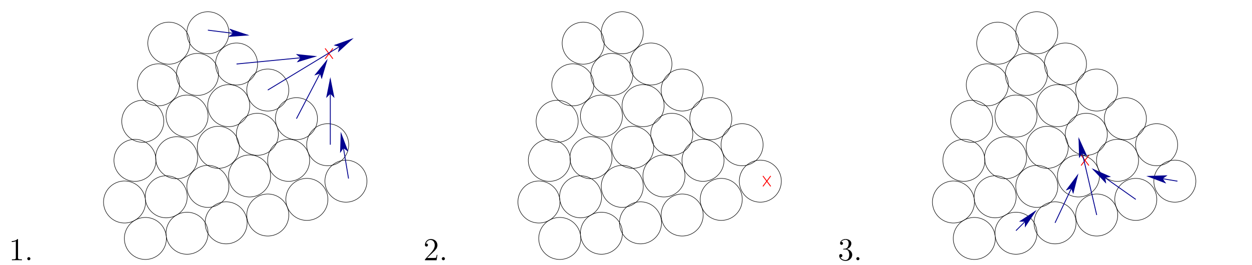


Figure 6: Three cases when applying forces
(attractive point 1. is out of the system; 2. lies on the skin; 3. is in the system)

3.3. Applied force

Once we have defined the constraints on the location and the direction of the force, we can define its amplitude. The force we want to apply must satisfy two other constraints:

1. it must not be stronger than the particle system's cohesion forces (see eq. (4)),
2. it must generate constant volume deformation.

The maximal force that a particle can support without being detached from the system is equal to the sum of all the maximal forces applied by its neighbours.

In the case of a mono-layer particle system (all the particle having the same size), the sum should not exceed the value of $W(i)F_{max}/2$, with $W(i)$ the number of neighbors of the particle i and F_{max} the maximal cohesion force. Applying the forces only to the particles next to the attractive points will increase the simulation time, because of the system's inertia. Hence, we equally apply forces to all neighboring particles. However, with this kind of forces, care should be taken so that the epicenter particle is the most attracted and the others are attracted decreasingly attracted according to their distance (see Fig. 7).

When the particle reaches its destination (*particle* i touches the attractive point) the force should be zeroed.

In Fig. 7, r_0 is the particle distance (radius) at which the force must always be null, d_{min} is the distance between the epicenter particle and its associated attractive point. Eq.

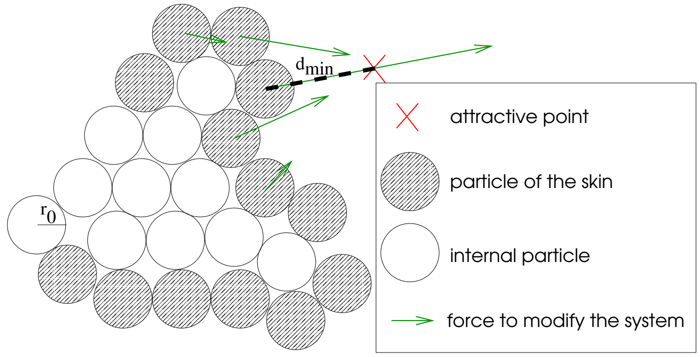


Figure 7: Attractive force form and behavior at a given time

(5) represents our defined deformation force applied to skin particles. This force is a kind of modified mass/spring force.

$$\vec{G_{ik}} = K_{r_k} \left(\frac{d_{min_k} - r_0}{r_{ik} - r_0} \right)^{\alpha_k} (r_{ik} - r_0) \frac{\vec{r_{ik}}}{r_{ik}} - K_a \vec{v_i} \tag{5}$$

In this equation,

- $\vec{G_{ik}}$ is the force between the particle i and its attractive point k;
- $\left(\frac{d_{min_k} - r_0}{r_{ik} - r_0}\right)^{\alpha_k}$ is the term that prevents detachment and permits to attract mostly the epicenter particle;
- $K_{r_k}(r_{ik} - r_0)$ is the elasticity term permitting to define the amplitude of the applied forces;
- d_{min_k} is the distance between the epicenter particle and its attractive point k;
- r_{ik} is the distance between the particle i and the attractive point k;
- α_k is a parameter permitting to diminish non linearly the amplitude of the applied forces to neighbors of the epicenter particle for attractive point k.
- K_a is a damping coefficient and $\vec{v_i}$ represents the velocity of the particle i.

We determine K_{r_k} by solving (5) for the maximal force that can be applied to the epicenter particle, that means when $r_{ik} = d_{min_k}$ (see eq. (6)):

$$K_{r_k} = \frac{F_{max_k}}{d_{min_k} - r_0} . \tag{6}$$

3.4. Results

Fig. 8 shows how a cube behaves while we try to elongate it to two attractive points. In Fig. 9, we try to predict a prostate's deformation between two sessions of radiotherapy. In this case we have used 850 particles and 87 attractive points. The simulation necessitates 15 minutes of computation using a PIII 500Mhz. In Fig. 10, we apply forces on a cube in order to fit lung dots sampled from its surface. In this case we have used 1000 particles and 220 attractive points within 15 minutes of computation.

4. Conclusion

The results presented in this paper have been successfully used in a medical project. The advantage of our approach is to permit the complete reconstruction of a shape from unorgan-

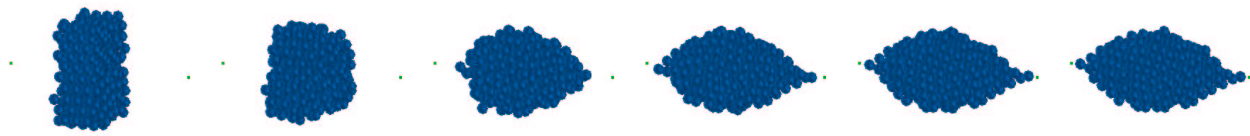
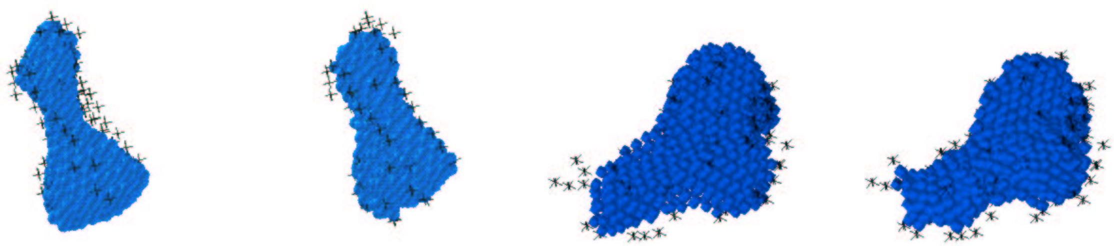


Figure 8: Sequence of elongating a cube



a. b. c. d.

Figure 9: Side view (a. original shape b. deformed) and front view (c. original shape d. deformed) of prostate fitting

ised partial data. However, the computation time is rather long. To overcome this problem, there are several points in our algorithms that can be optimized:

1. the calculation of the particles of the skin and their neighbourhood area is the most time consuming in our algorithm. It may be interesting to experiment other kinds of updating forces;
2. the complexity of the algorithm of the cohesion force calculation is actually $O(n^2)$ and we are working to reduce it to $O(nm)$ (with $m << n$) by using shorter range forces;
3. we have not yet applied our updating method to a multi-layer particle system which can permit to obtain a faster convergence [7, 5].

As the particles keep their volume constant and remain in contact with each other, the interstitial volumes between particles do not vary too much so that the volume change during the deformation can be neglected. Hence "volume preserving" morphing is a possible application of that method.

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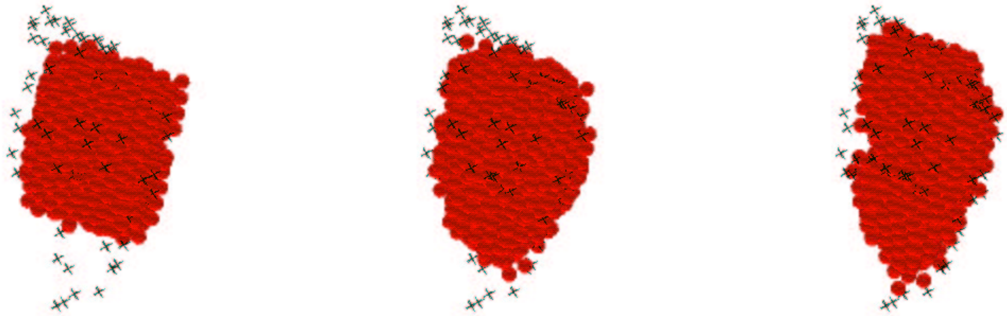

Figure 10: Morphing a cube into a lung

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