FRACTAL COMPRESSION OF IMAGES WITH PROJECTED IFS

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Introduction

Our goal

- Obtain compact descriptors for images
- Threat both smooth and rough iamges
- Data entries: grey-level images

Our approach

- Deterministic model: projected IFS attractors
- Combination of models in a quadtree structure
- Non-linear fitting problem
- Variable accuracy
- Resolution with LEVENBERG-MARQUARDT algorithm
- Optimisation with a method based on Ramchandran and Vetterli algorithm

The model

Overview

- Fractal model: Iterated Function Systems
- CAGD model: free forms
- **Projected IFS attractors**

IFS Formalism

- Used to describe fractal objects
- Based on self-similarity
- An object is described by a set of contractions of a metric space E:

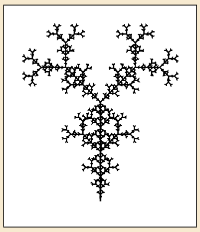
$$\mathbb{T} = \{T_0, \ldots, T_N - 1\}$$

- The attractor $\mathcal{A}(\mathbb{T})$ of $\mathbb{T}$:

$$\mathcal{A}(\mathbb{T}) = \lim_{n\to\infty} \mathbb{T}^n K$$

where K is a random compact.

IFS example in $\mathbb{R}^2$



$$\begin{aligned} T_0 &= S(0.5) \\ T_1 &= T(0,0.5)S(0.5) \\ T_2 &= T(0,1)R(\pi/4)S(0.5) \\ T_3 &= T(0,1)R(-\pi/4)S(0.5) \\ \mathcal{A}(\mathbb{T}) &= T_0\mathcal{A}(\mathbb{T}) \cup T_1\mathcal{A}(\mathbb{T}) \cup T_2\mathcal{A}(\mathbb{T}) \cup T_3\mathcal{A}(\mathbb{T}) \end{aligned}$$

Projected IFS attractors

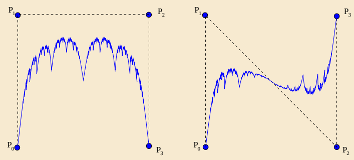
Formalism

- The metric space E is a barycentric space
- T_i are contractive matrices with barycentric columns
- A set of control points $P = (p_0, \ldots, p_n)$ is used to perform a projection
- Each point of $\mathcal{A}(\mathbb{T})$ is projected:

$$P\mathcal{A}(\mathbb{T}) = P \lim_{n\to\infty} \mathbb{T}^n K$$

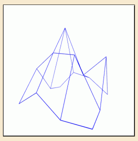
Curve example

- Deformation of a curve using the control points

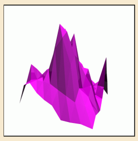


Model features

A simplified construction algorithm

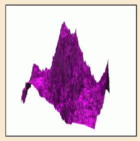


Control grid



Step 1

...



Step 4

Parametric function representation

- BARNSLEY introduces an address function by indexing transformations with a finite set $\Sigma = \{0, \ldots, N-1\}^a$:

$$\sigma \in \Sigma^\omega \mapsto \phi(\sigma) = \lim_{n\to\infty} T_{\sigma_1} \ldots T_{\sigma_n}\lambda$$

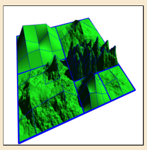
- It is easy to construct a parametric representation:

$$\Phi(s) = \phi(\sigma) \text{ with } s = \sum_{i=1}^{\infty} \frac{1}{N^i}\sigma_i$$

- And the projected attractor has a function representation:

$$\begin{aligned} F(s) &= P\Phi(s) \\ &= P\phi(\sigma) \\ &= P\lim_{n\to\infty} T_{\sigma_1} \ldots T_{\sigma_n}\lambda \end{aligned}$$

Combination of models in a quadtree structure



Example of a combination of models in a quadtree structure

- Each leaf of the quadtree contains a complete model $(P_\gamma, \mathbb{T}_\gamma)$
- This implies boundary constraints
- These constraints and inner constraints on a single model are expressed in a common formalism

[a] For curves. For surfaces $\Sigma = \{0, \ldots, N-1\}^2$ is used

Approximation Method

Approximation problem formalism

- Given an image $\mathbf{Q}_{ij\,(i=0,\ldots,N^p\; j=0,\ldots,N^p)}$
- Find the projected IFS attractor that minimises:

$$\sum_{ij} ||\mathbf{Q}_{ij} - F(\frac{i}{N^p}, \frac{j}{N^p})||^2$$

- A reduced number p of iteration is needed

$$\sigma = \sigma_1 \ldots \sigma_p 00 \ldots$$

$$F(\frac{i}{N^p}, \frac{j}{N^p}) = PT_{\sigma_1\tau_1} \ldots T_{\sigma_p\tau_p} e_{00}$$

Non-linear fitting

- Parameterisation of the model: parameter vector $\mathbf{a}$
  - Matrix coefficients
  - Control points coordinates
- Fractal familly function $F_\mathbf{a}$ issued from the couple $(P_\mathbf{a}, \mathbb{T}_\mathbf{a})$
- Approximation → non-linear fitting

$$\mathbf{a}_{opt}(\mathbf{Q}) = \underset{\mathbf{a}}{\operatorname{argmin}}\, d(F_\mathbf{a}, \mathbf{Q})$$

- Resolution provided by the LEVENBERG-MARQUARDT algorithm

Optimisation

- Exhaustive approximation of image parts using a recursive quadtree refinement
- A large number of images descriptors
  - Several models
  - Several quantificators
    - For control points (P)
    - For transformations ($\mathbb{T}$)
- Lagrange multiplier formalism

$$J(\lambda, R_i, D_i) = D_i + \lambda R_i$$

- Method based on Ramchandran and Vetterli (1993)

Results

Compression of grey-level images

Compression rate: $0.044bpp$

Original image

Image compressed with JPEG2000, PSNR $= 23.1dB$

Image compressed with PIFS, PSNR $= 24.7dB$

Compression rate: $0.064bpp$

Original image

Image compressed with JPEG2000, PSNR $= 21.4dB$

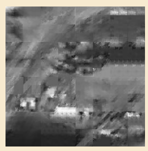
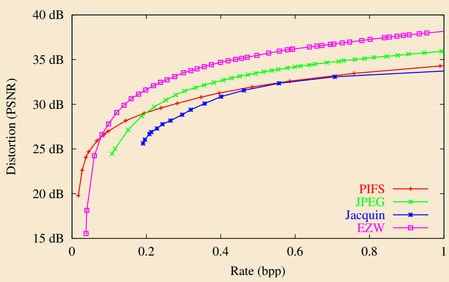
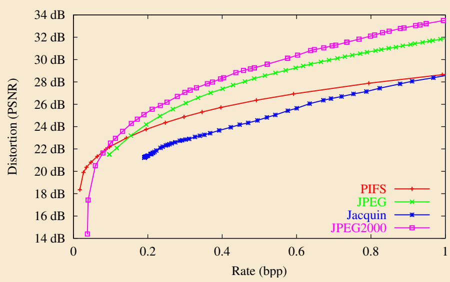


Image compressed with PIFS, PSNR $= 21.4dB$

Rate/Distortion curve





Conclusion

- Better than standard fractal image compression
- Same or better than JPEG2000 in the very low bitrate context