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Fractal coding of shapes based on a projected IFS model

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Goals

- Descriptor for 2-D curve shapes
- Handle both rough and smooth curves
- Data entries: a sampled curve composed of $m+1$ points

$$\mathbf{Q} = Q_0, \ldots, Q_m$$

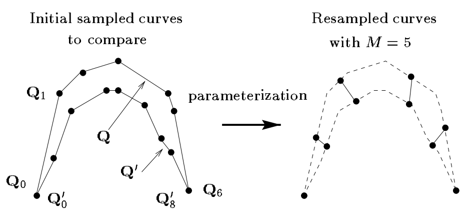
Our approch

- Projected IFS model
- Non linear fitting formalism
- LEVENBERG-MARQUARDT resolution

Approximation criteria

- Uniform parameterization $\mathcal{G}_{\mathbf{Q}}$ and $\mathcal{G}_{\mathbf{Q}'}$ of the two curves $\mathbf{Q}$ and $\mathbf{Q}'$
- Computation of the distance:

$$\mathcal{D}(\mathbf{Q},\mathbf{Q}') = \sum_{i=0}^{M} \left[d\left(\mathcal{G}_{\mathbf{Q}}(\frac{i}{M}), \mathcal{G}_{\mathbf{Q}'}(\frac{i}{M}) \right) \right]^2$$



Resolution

- Mapping between coefficients of the model and a parameter vector $\mathbf{a}$

$$T_0(\mathbf{a}) = \begin{pmatrix} 1 & a_1 & a_4 & a_7 \\ 0 & a_2 & a_5 & a_8 \\ 0 & a_3 & a_6 & a_9 \\ 0 & 1-(a_1+a_2+a_3) & 1-(\ldots) & 1-(\ldots) \end{pmatrix}$$

- Fractal curve familly $\mathcal{Q}(\mathbf{a})$
- The approximation problem is to find the optimal parameter vector $\mathbf{a}_{\text{opt}}$:

$$\mathbf{a}_{\text{opt}} = \arg\min_{\mathbf{a}} \sum_{i=0}^{M} \left[d\left(\mathcal{G}_{\mathbf{Q}}(\frac{i}{M}), \mathcal{G}_{\mathcal{Q}(\mathbf{a})}(\frac{i}{M}) \right) \right]^2$$

- LEVENBERG-MARQUARDT algorithm is applied to solve the problem

Fractal model

❶ Iterated Function System

- Set of contractive operators

$$\mathcal{I} = T_0, \ldots, T_{N-1}$$

- An attractor can be associated

$$\mathcal{A}(\mathcal{I}) = \lim_{n\to\infty} \mathcal{I}^n K$$

❷ Projected IFS

- A set of control points is added

$$P = p_0, \ldots, p_n$$

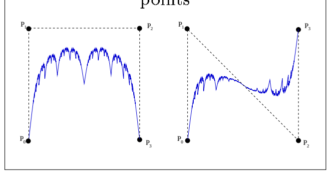
- Projected IFS attractor

$$P\mathcal{A}(\mathcal{I}) = \{P\lambda \mid \lambda \in \mathcal{A}(\mathcal{I})\}$$

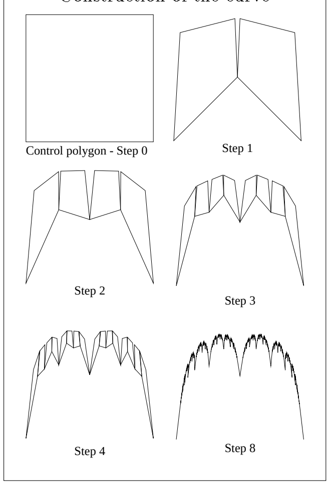
❸ Advantages

- Control points P allow a global deformation
- IFS gives a fractal aspect

Global deformation with control points

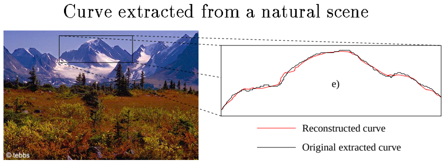


Construction of the curve



Results

Curve extracted from a natural scene



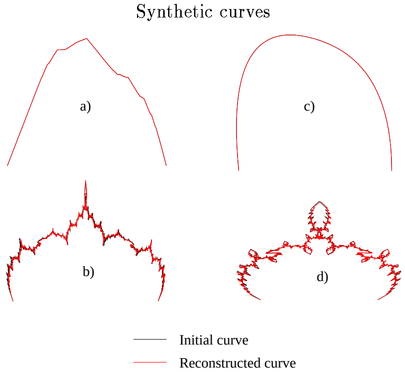
Associated model

$$T_0 = \begin{pmatrix} 1.000 & 0.232 & 0.056 & 0.297 \\ 0.000 & 0.768 & 0.081 & 0.699 \\ 0.000 & 0.000 & 0.366 & 0.219 \\ 0.000 & 0.000 & 0.498 & -0.215 \end{pmatrix}$$

$$T_1 = \begin{pmatrix} 0.297 & 0.437 & 0.000 & 0.000 \\ 0.699 & 0.093 & 0.000 & 0.000 \\ 0.219 & -0.008 & 0.543 & 0.000 \\ -0.215 & 0.478 & 0.457 & 1.000 \end{pmatrix}$$

$$P = \begin{pmatrix} -1.000 & -1.596 & 7.430 & 1.000 \\ 1.000 & 2.885 & 2.540 & 1.000 \end{pmatrix}$$

Synthetic curves



Curve information

Figure	Number of points	Total number of bits associated to the original curve: B	Entropic coding of B (bits)
a	257	16448	8304
b	1025	65600	30960
c	513	32832	15600
d	1025	65600	26248
e	210	13440	5432

Compression ratio / distortion

Figure	Total number of bits for coding the projected IFS model	Compression ratio	Distortion χ^2
a	368	22.6	$1.1e^{-5}$
b	368	84.1	0.078
c	368	42.4	$1.4e^{-5}$
d	368	71.3	0.058
e	368	14.8	0.0146