Fractal coding of shapes based on a projected IFS model

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Abstract

This paper addresses the problem of approximation of natural complex shapes. Using MPEG-7 terminology, this problem can be considered as a descriptor search for a shape feature. This shape can be defined either as a frontier between image regions or a natural curve. For this purpose, an original descriptor which combines an Iterated Function System (IFS) model and the notion of free form curves is proposed. A set of control points permits to define the IFS model in a barycentric space. This generalization adds a real flexibility to fractal approximation techniques enriching the set of contractive operators which are candidate to model the self-similarity. This new descriptor named projected IFS model allows the reconstruction of a shape using a projection via the control points. It is adapted to the representation of both smooth shapes (man-made objects, body,..) and fractal shapes (mountain, cloud, tree,..). Results on synthetic shapes and a real mountain shape are presented.

1 Introduction

The approximation of natural complex shapes constitutes an important research area to bring solutions to reconstruction and representation problems for several application domains (medical imaging, multimedia data representation, Computed Assisted Design).

In fractal theory, the determination of an Iterated Function System (IFS) model for approximating natural data, is called "the inverse problem". The fractal image coding techniques first introduced by [3] constitute an efficient example for this approach.

Fractal approximation techniques, although the advantage of describing self-similar objects, suffer from an important drawback consisting in a lack of control on the fractal figures to describe. This is essentially due to the use of the contractive affine operators defined in a reduced space, i.e. $\mathcal{X} = \mathbb{R}^2$ for images. In contrast, free form approximation methods allow to control the objects with high flexibility via a set of control points. But only the smooth objects are reconstructed.

The proposed shape descriptor unifies the advantages of these two strategies and aims to approximate the natural complex shapes with an IFS model coupled to a set of control points. In this way, these control points lead to a generalization of the IFS model defined in $\mathcal{X} = \mathbb{R}^n$ space where we search the contractive opeators for the self-similarity description. The shape can be recontructed by using a projection via the control points. This new descriptor is named projected IFS model.

2 Description model

Introduced by Barnsley[1] in 1988, IFS (Iterative Function System) technique permits to generate a geometrical shape or an image with an iterative process. An IFS-based modeling system is defined by a triple $(\mathcal{X}, d, \mathcal{S})$ where :

- $(\mathcal{X}, d)$ is a complete metric space, $\mathcal{X}$ is called *iteration space*;
- $\mathcal{S}$ is a semigroup acting on points of $\mathcal{X}$ such that : $p \to Tp$ where T is a contractive operator, $\mathcal{S}$ is called *iteration semigroup.*

An *IFS (Iterative Function System)* is a finite subset of $\mathcal{S}$: $\mathcal{I} = \{T_0, ..., T_{N-1}\}$ with $T_i \in \mathcal{S}$. We note $\mathcal{H}(\mathcal{X})$ the set of non-empty compacts of E. The associated Hutchinson operator is :

$$K \in \mathcal{H}(\mathcal{X}) \quad \mapsto \quad \mathcal{I}K = T_0 K \cup ... \cup T_{N-1} K$$

This operator is contractive in the complete metric space $\mathcal{H}(\mathcal{X})$ and admits a fixed point, called *attractor*, defined by [1] :

$$\mathcal{A}(\mathcal{I}) = \lim_{n \to \infty} \mathcal{I}^n K \ with \ K \in \mathcal{H}(\mathcal{X})$$

So, each IFS $\mathcal{I}$ define an unique compact, $\mathcal{A}(\mathcal{I})$ is called the IFS attractor. With good hypothesis on IFS [5], the attractor is a curve : $\mathcal{A}(\mathcal{I}) = \{\phi(t) | \ t \in [0,1]\}$ where ϕ is a function $\phi : [0,1] \to \mathcal{X}$. Classicaly, in fractal compression [3] or fractal interpolation [1], $\mathcal{X} = \mathbb{R}^2$ and $\mathcal{S}$ is the semigroup of contractive affine operators. In our approach, we take an other space, the barycentric space $\mathcal{X} = \mathcal{B}_+^I$ defined by :

$$\mathcal{B}_+^I = \{(\lambda_j)_{j=0,\dots,n} \,|\, \lambda_j \geq 0 \,, \sum_{j=0}^{n} \lambda_j = 1\}$$

The iteration semigroup $\mathcal{S} = S_I$ is constituted by MARKOV matrices :

$$S_I = \{T \mid \sum_{j=0}^{n} T_{ij} = 1\,,\, T_{ij} \geq 0\ \forall i = 0, \ldots, N\}$$

This choice allows to define a generalization of IFS attractors named **projected IFS attractors** : $P\mathcal{A}(\mathcal{I}) = \{P\lambda \mid \lambda \in \mathcal{A}(\mathcal{I})\}$ where P is a sequence of control points : $P = (p_0, \ldots, p_n)$ and $P\lambda = \sum_{i=0}^{n} \lambda_i p_i$. In this way, we can construct a fractal curve characterised by a set of control points P [6] [5] [4] using the projection :

$$Q(t) = \sum_{i=0}^{n} \phi_i(t)\, p_i = P\phi(t)$$

where $\phi(t)$ is a vector of functions : $\phi(t) = (\phi_0(t), \ldots, \phi_n(t))^T$. Note that ϕ functions have a self-similarity structure. This implies a simple relation between ϕ_i and T_i :

$$\phi(\tau_i\, t) = T_i\, \phi(t)$$

where $\tau_i : [0, 1] \rightarrow [0, 1]$ is a subdivision operator.

3 Determination of IFS model and the control points

We have to find the description model for a given curve. The approach we propose is based on the visualization algorithm (fractal object synthesis). Therefore, this direct approach will be detailed first. Then, we will show how we reverse the process to solve the inverse problem.

3.1 The visualization algorithm (the decoding process)

The visualization process is recursive. First, the control polygon defined by a given set of control points is used to initialize this process. Then, at each step, the N contractive operators $\{T_0, \ldots, T_{N-1}\}$ are applied to all the polygons. Though, the number of polygons is multiplied by the factor N at each step of the visualisation algorithm. The figure 1 shows this principle.

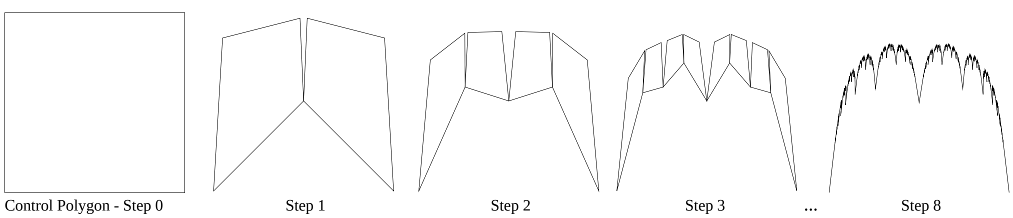

Figure 1: Example of visualization (4 control points and 2 transformations).

3.2 Descriptor construction (the coding process)

The visualization algorithm has to be reversed. For a given curve, we try to wrap the curve up with tangents[2] (Figure 2a). Then, this curve is divided in order to apply the same algorithm to each sub-curve. That way, the method is recursive as the visualization process. Finally, a set of control polygons is constructed. To find out the contractive operators $\{T_0, \ldots, T_{N-1}\}$ corresponding to the original curve, we just have to create a set of linear equations involving the transformations, the curve samples and the control points early determined. The last step is the resolution of this system that is provided by a least-square approach. The figure 2a illustrates the “tangent” method. The initial curve

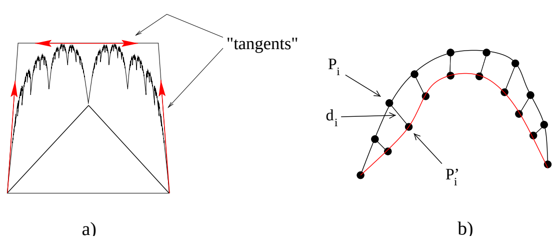

Figure 2: The “tangent” method (a) and distance between curves (b).

and the reconstructed curve are resampled in order to measure a distance between these curves (see figure 2b). Figure 4c presents the set of control points P and the IFS model $\{T_0, T_1\}$ obtained for an original curve (figure 4b).

Curver number	Number of points	Normal coding	Fractal coding	Compression ratio	Mean Distance
1	210	3360 bits	368 bits	9	0.054210
2	4000	32000 bits	368 bits	87	0.141681
3	513	4104 bits	368 bits	11	0.044798

Table 1: Numeric results

3.3 Results and conclusion

Figures 3 and 4 give examples of reconstruction with natural and synthetic curves respectively. The visual quality is satisfactory. In Table 1, information about these curves, the total number of bits required for coding the complete model $\{P, T_0, T_1\}$, the compression ratio and the mean square distance is given. Ongoing works are the generalization of the projected IFS model to contours and surfaces.

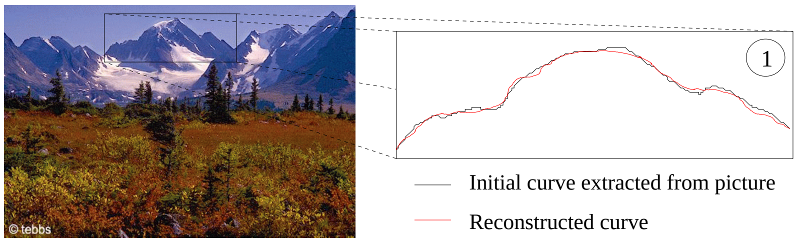

Figure 3: Example of result on a curve extracted from a natural picture.

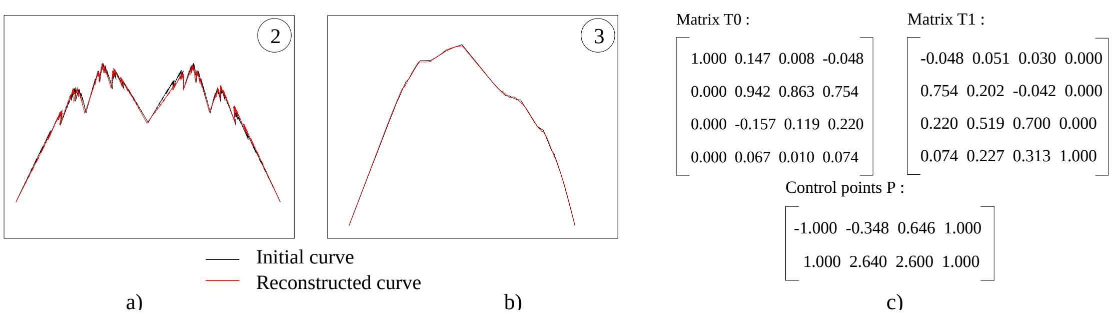

Figure 4: Two examples of results on synthetic curves and the projected IFS model $\{P, T_0, T_1\}$ for Curve 3.

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