Fractal approximation of surfaces based on projected IFS attractors

EuroGraphics 2001 - Short Presentation

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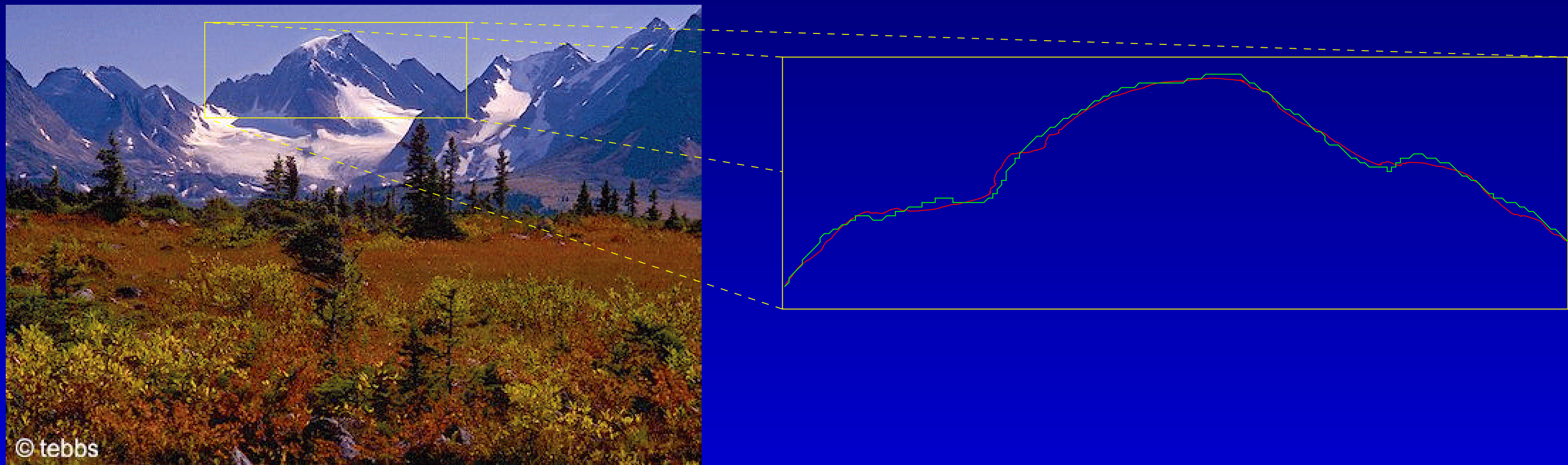
Goals

❶ Modelisation of rough objects

❷ Through approximation

❸ Example: approximation of a curve extracted from a picture



Introduction

❶ Rough modeling

- Random models
- Approximation is impossible

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- ❶ Rough modeling
  - Random models
  - Approximation is impossible

- Our method
  - ➠ Rough model but deterministic
  - ➠ Calculable function
  - ➠ Parameterisation of the model allows an approximation

Our model: fractal aspect

- IFS formalism
  - Describes fractal objects
  - Based on self-similarity

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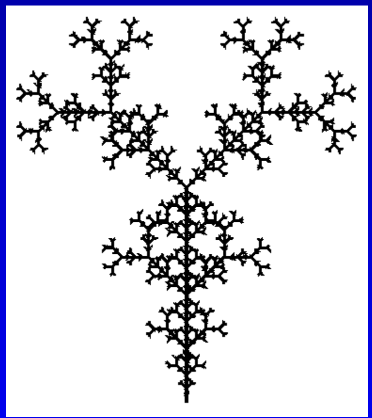
- Example : in $\mathbb{R}^2$

$$A = T_0 A \cup T_1 A \cup T_2 A \cup T_3 A$$



$$\begin{aligned} T_0 &= S(0.5) \\ T_1 &= T(0,0.5)S(0.5) \\ T_2 &= T(0,1)R(\pi/4)S(0.5) \\ T_3 &= T(0,1)R(-\pi/4)S(0.5) \end{aligned}$$

Our model: formalism

- IFS : $I = \{T_0, \ldots, T_{N-1}\}$
  - With T_i contractions of a metric space E
  - IFS attractor

$$\mathcal{A}(I) = \lim_{n \to \infty} I^n K$$

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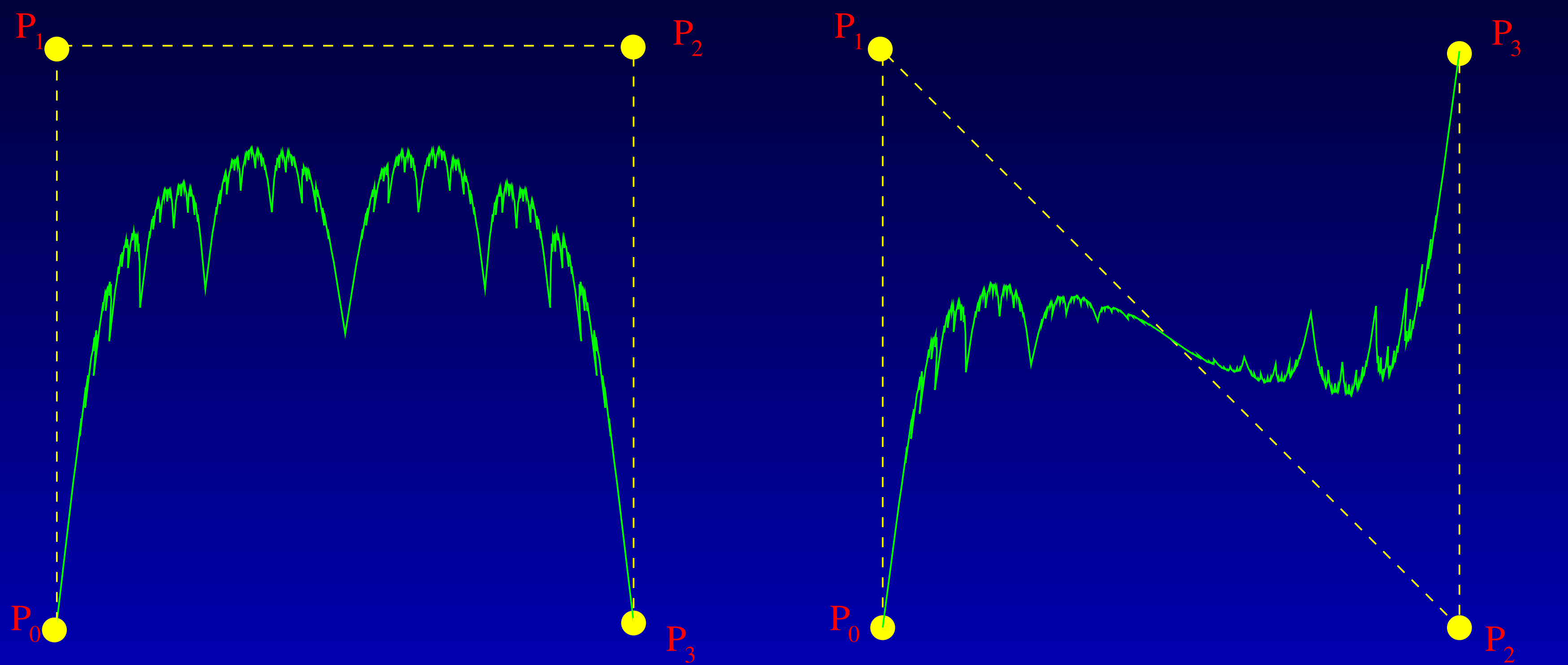
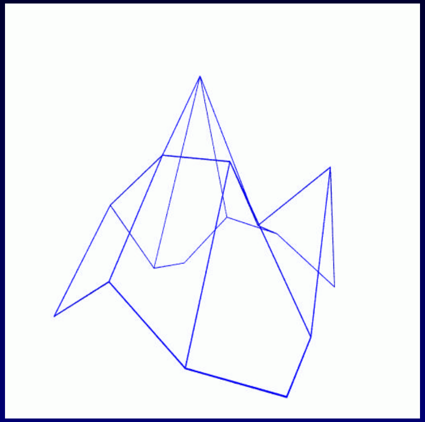
- Fractal free-forms
  - E is a barycentric space, T_i are barycentric columns matrices
  - Projection through control points

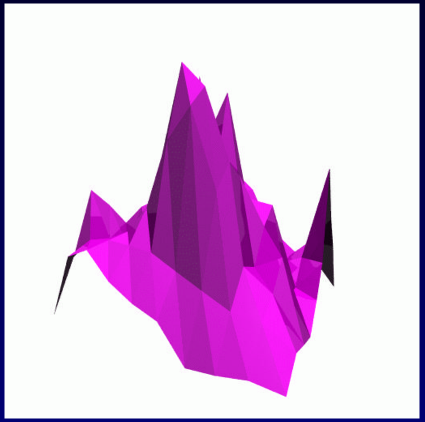
$$P\mathcal{A}(I) = P \lim_{n \to \infty} I^n K$$

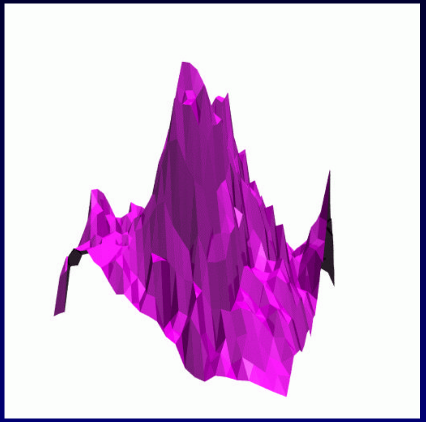
Curve deformation example

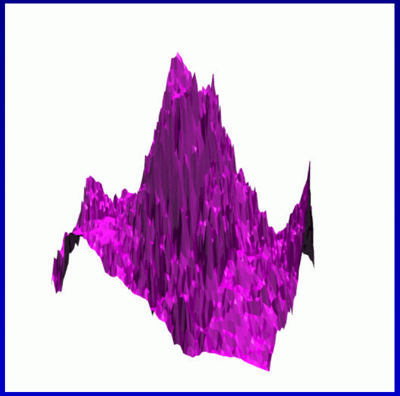


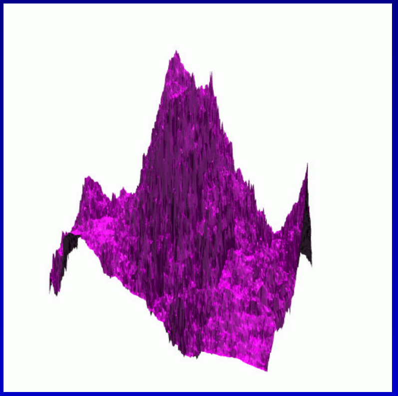
Fractal surface construction


Control grid


Step 1


Step 2


Step 3


Step 4

Approximation method

- Parameterisation of the model: parameter vector **a**
  - ➠ Matrix coefficients
  - ➠ Control-point coordinates

- Surface family

$$F_{\mathbf{a}} = P_{\mathbf{a}} \mathcal{A}(I_{\mathbf{a}})$$

- Approximation ➠ non-linear fitting

$$\mathbf{a}_{\mathrm{opt}}(\mathbf{Q}) = \underset{\mathbf{a}}{\operatorname{argmin}}\, d(F_{\mathbf{a}}, \mathbf{Q})$$

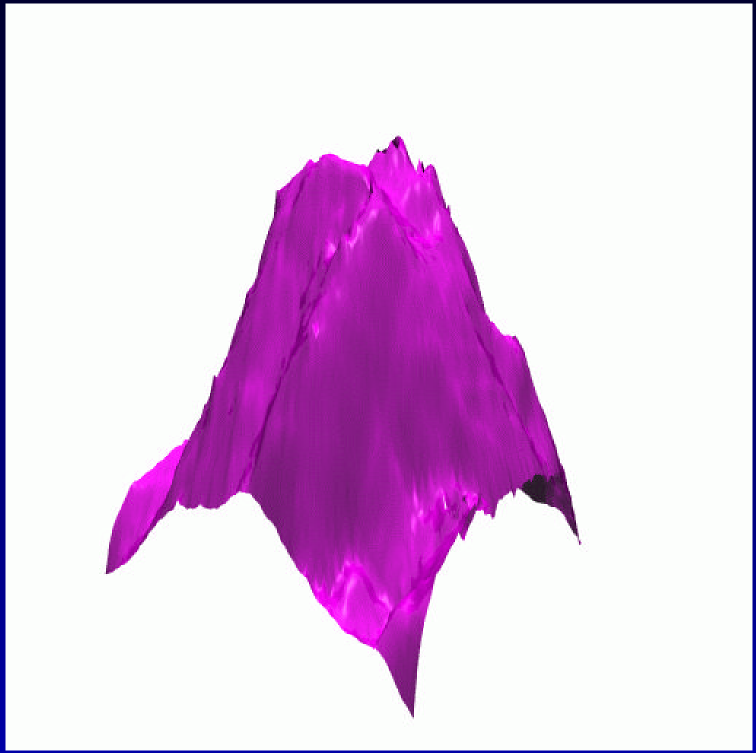
- Resolution: Levenberg-Marquardt

Results (smooth surface)

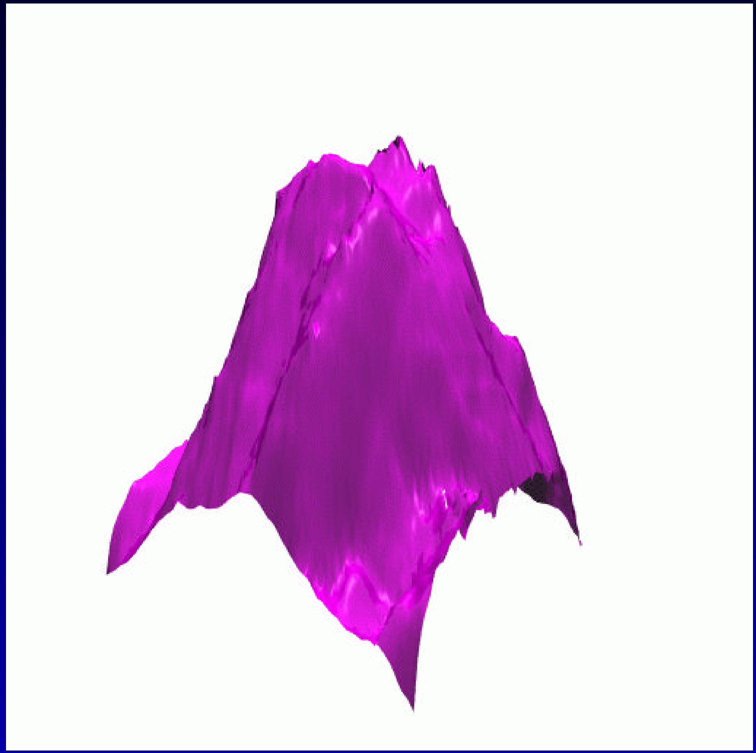
Original surface

Approximated surface

Results (synthetic surface)

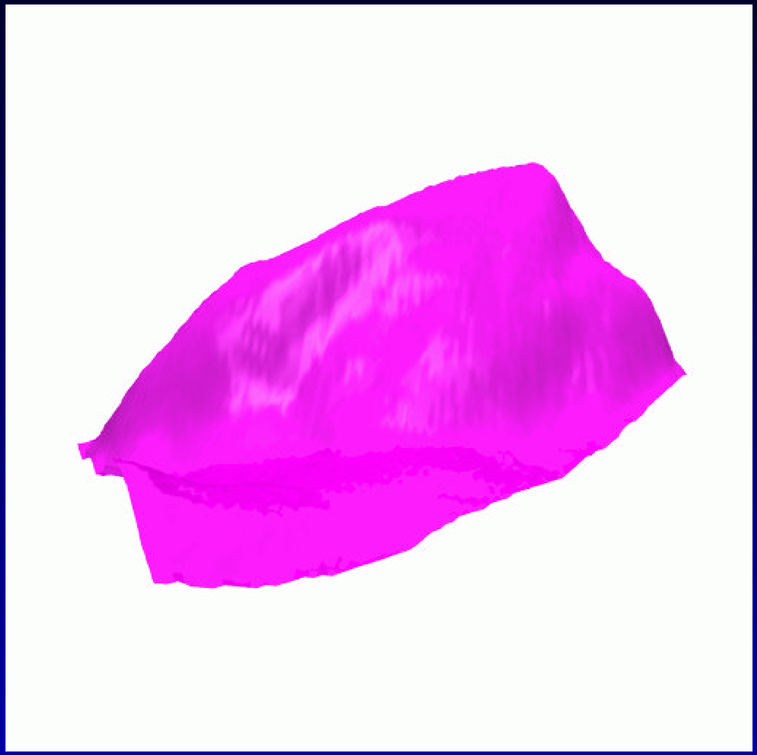


Original surface

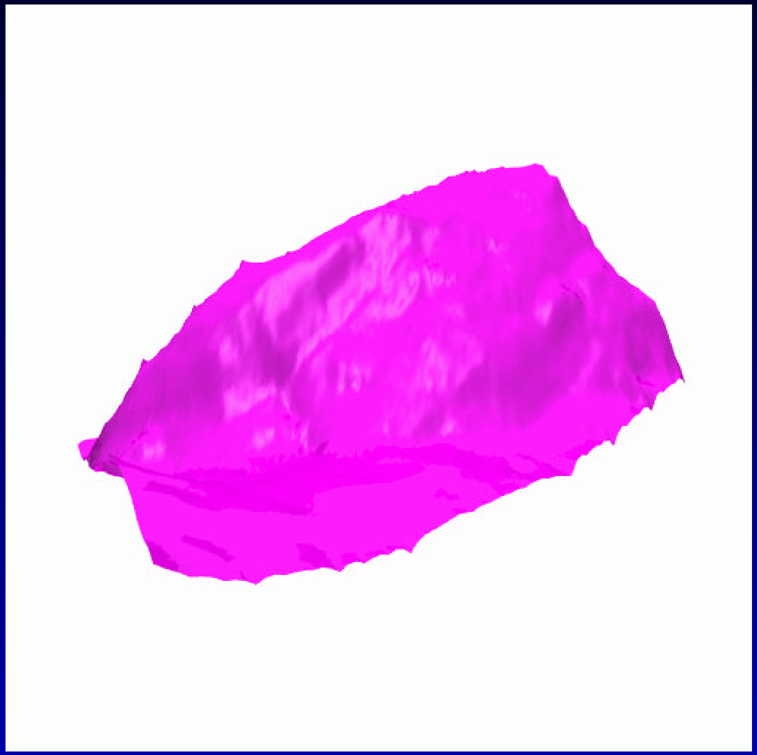


Approximated surface

Results (geological surface)



Original surface



Approximated surface

Conclusion

❶ Advantages

- Large modelisation area
- General approximation method

Conclusion

1. Advantages
   - Large modelisation area
   - General approximation method
2. Encountered problems
   - Compromise computing time / model complexity
   - Difficulties for real 3D models

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❶ Advantages

- Large modelisation area
- General approximation method

❷ Encountered problems

- Compromise computing time / model complexity
- Difficulties for real 3D models

- Ongoing work
  - ➠ Test on real data
  - ➠ More flexible model