

Digital Geometry in a Nutshell

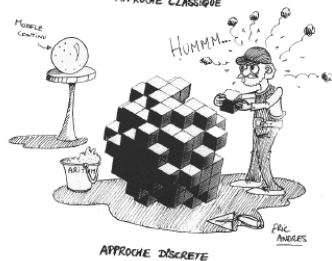
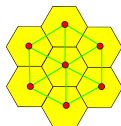
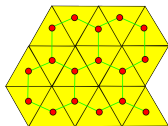
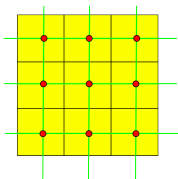
David Coeurjolly

Laboratoire d'InfoRmatique en Image et Systèmes d'information

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<http://liris.cnrs.fr>

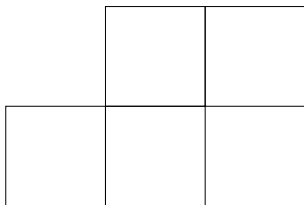
Discrete Geometry in one slide

Analysis of geometrical problems on objects defined on regular grids



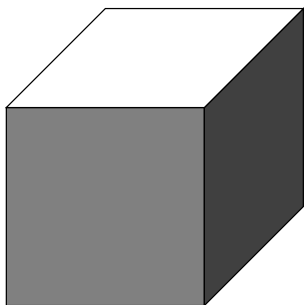
[Eric Andres 95]

Digital Geometry Ecosystem



Digital straight segment in \mathbb{Z}^2 ...

Digital Geometry Ecosystem (bis)



Digital sphere in \mathbb{Z}^3 ...

Motivations

Pragmatic approach (*data driven*)

- ≡ Data produced by acquisition devices which consider an underlying grid (CDD, Scanner, Scanner+T, ...)
- ≡ Modeling of numerical problems on grids/integer numbers
- ≡ [BERNOUILLI, ROSENFELD, ...]

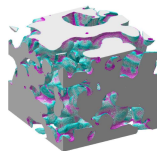
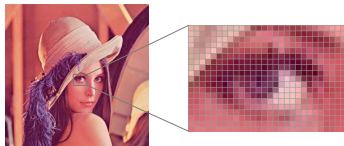
⇒ *Arithmetization*

Constructive approach (*model driven*)

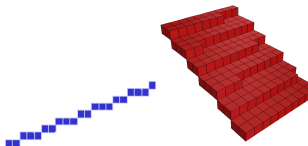
- ≡ Construction from scratch of a geometry based on integer numbers
- ≡ e.g. Theory based on the Non-Standard Analysis
- ≡ [HARTONG, REEB, REVELLES, ...]

⇒ *Modeling*

⇒ Complete Geometrical Paradigm (objects, axioms, ALGORITHMS, ...) well-adapted to image analysis



[Météo-France/ESRF]



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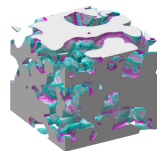
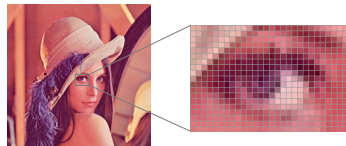
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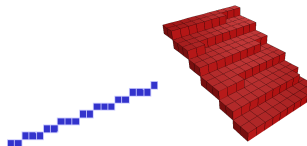
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[Météo-France/ESRF]



Euclidean Geometry vs. Digital Geometry

First Euclid's axiom

Given two points A and B ($A \neq B$), there exists a unique straight line containing A and B .



Euclidean Geometry vs. Digital Geometry

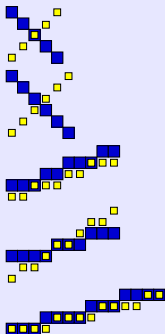
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Euclidean Geometry vs. Digital Geometry

Intersection between straight lines



1 single point

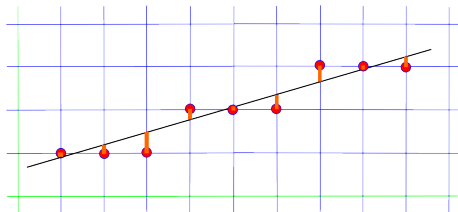
no intersection even digital straight lines are not parallel

parallel digital straight lines may share points

connected set of points

disconnected set of points

Arithmetic rocks



Let $d : y = \alpha x + \beta$ be straight line in the first octant and $D = \mathcal{D}_{\mathbb{G}}(d)$ its digitization, then $D = \{p_i\}$ such that $x_i = n$ and $y_i = \lfloor \alpha x_i + \beta + 0.5 \rfloor$ with $n \in \mathbb{Z}$

Theorem

α is a rational fraction (i.e. $\alpha = \frac{p}{q}$ with $p, q \in \mathbb{Z}$) \Rightarrow the DSS freeman code of D is periodic

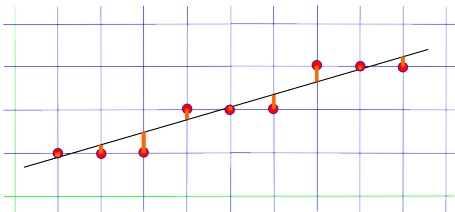
Hints

If $\alpha = \frac{p}{q}$, then the set of values $\alpha * n + \beta - \lfloor \alpha * n + \beta + 0.5 \rfloor$ is finite and periodic

Conversly

If α is irrational fraction, DSS freeman code of D is aperiodic

Arithmetic rocks



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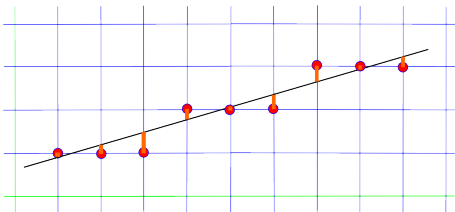
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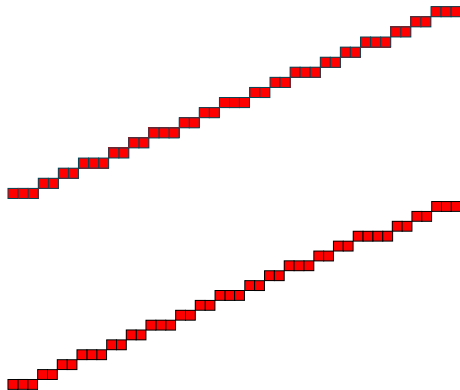
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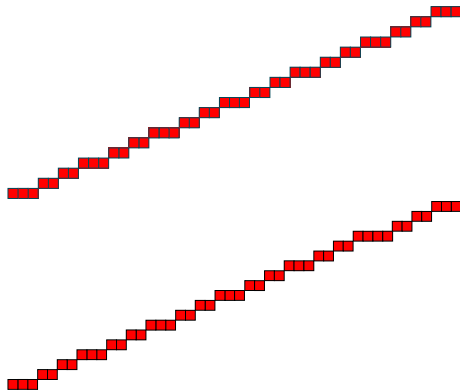
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Digital Straight Line Examples



the second one is not a DSS ;)

Digital Straight Line Examples



the second one is not a DSS ;)

Discrete Geometry Model

[Continuous world]



Discrete Analytic Models

Grid definitions, digitization schemes, fundamental algebraic and arithmetic facts, ...

Fundamental Objects and Properties

Points, straight lines, planes, simplexes, circles, intersection, parallelism, ...



Objects Analysis/Modeling

Feature extraction, distance transformation, ...

Loss-less Model Conversion

Reversible reconstruction with linear structures, volumic encoding with unions of balls, ...

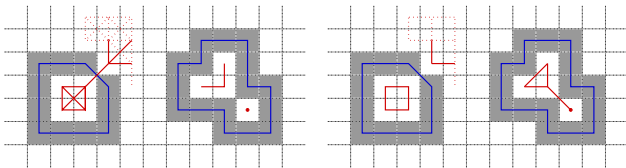


[Polygons, Meshes, Poly-ball structures]

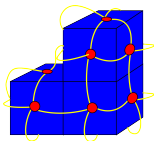
ALGORITHMIC POINT OF VIEW

Object and Contour Representation in Higher Dimension

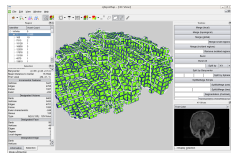
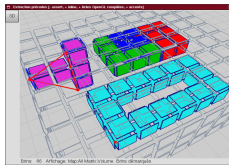
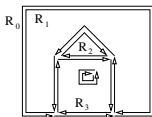
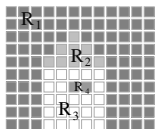
2D:



3D:



Regions in n-D images: *Topological Maps*



n-D Contour Analysis

Recognition of fundamental structures

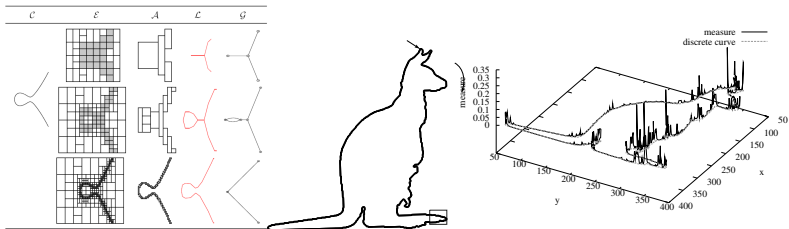
straight lines, circles, planes, ellipses,

Geometrical Reconstruction

Digital curve/surface \leftrightarrow Polygonal curve/surface

Shape Descriptor

- geometric estimator : length, curvature, . . .
- Convexity/concavity analysis
- Relationship between digital and continuous domains

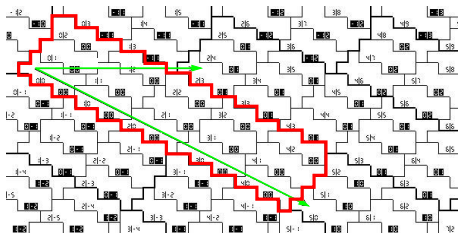


Arithmetization of Image Transformations

Back to Basics

How to transform an image with affine function ?

$$X \mapsto \frac{MX + V}{\omega} \quad \dots \quad X \mapsto \left\lfloor \frac{MX + V}{\omega} \right\rfloor$$



Rounding operator \Rightarrow Periodic structures \Rightarrow Paving pre-computation \Rightarrow fast algorithms !

High Performance Volumetric Analysis

Volumetric Analysis of $d - D$ images

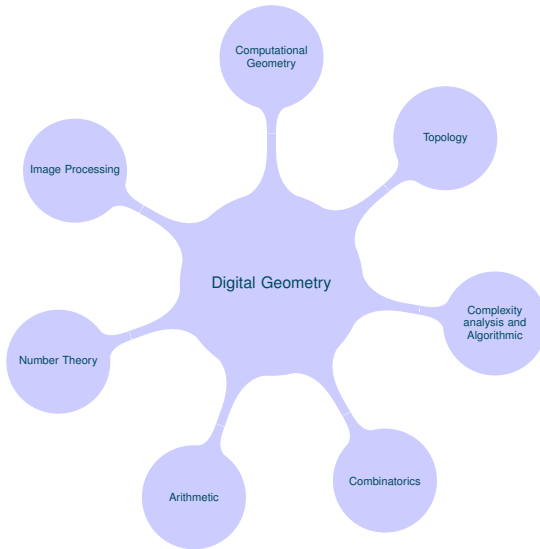
	Comp. Cost	sep. algorithm	Toric spaces	Multi-thread
EDT	$O(n^d)$	✓	✓	✓
multi-region EDT	$O(n^d)$	✓	✓	✓
REDT	$O(n^d)$	✓	✓	✓
RDMA	$O(n^d)$	✓	✓	✓
Voronoi/Power Diagram	$O(c \cdot n^d)$	✓	✓	✓
Local Thickness Computation	$O(n^d \log n)$	✓	✓	✓

Other projects

- ▣ Volumetric on Irregular Isothetic Grids
- ▣ Digital Sphere Tree

Demo

Conclusion



applied to image modeling/processing problems !