

# Document recto/verso separation with a dual layer Markov model

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# Plan

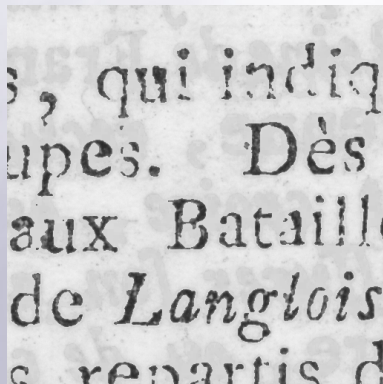
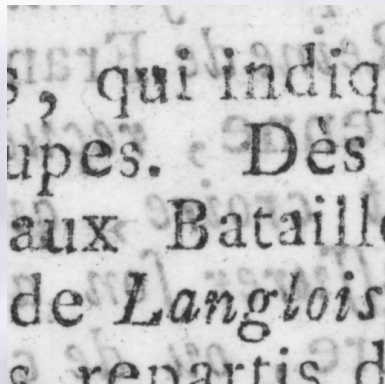
- 1 Introduction
- 2 The prior model
- 3 The observation model
- 4 Outline
- 5 Iterative graph cuts - the posterior probability and its maximization
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# The problem

**Blind** ink bleed-through removal (scans of the verso side are **not** available).

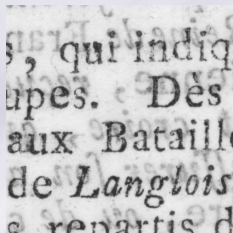


The verso contents must be replaced by background.

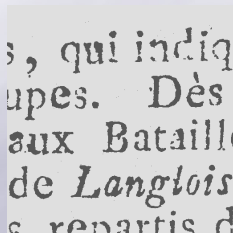
# Ink bleed-through removal

Problem : **image segmentation**. The classification decision for each pixel is based on

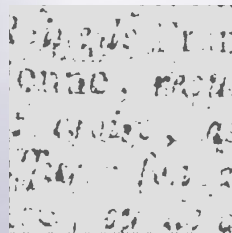
- ≡ the color/gray value of each pixel
- ≡ the **local structure** of the image.



input



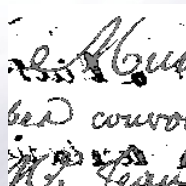
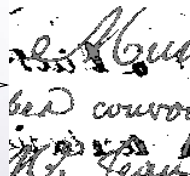
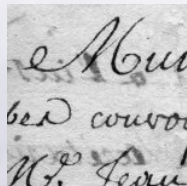
recto labels



verso labels

The framework : Bayesian estimation with two hidden Markov random fields (MRF)

# Bayesian estimation

 $P(f)$ 

 $P(d|f)$ 


PRIOR KNOWLEDGE

LIKELIHOOD OF THE DATA  
GIVEN THE HIDDEN VARIABLES

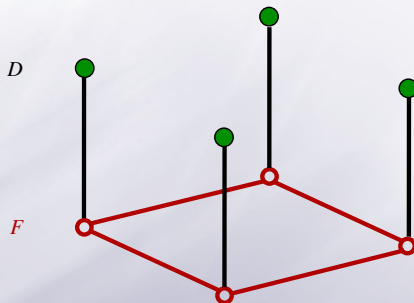
ESTIMATION

$$\hat{f} = \arg \max_f p(f|d)$$

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# A classical hidden Markov random field



Observed  
variable



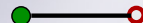
Hidden  
variable



Clique of the  
Prior model



Clique of the  
observ. model



$$P(f) = \frac{1}{Z} \exp \{-U(f)/T\}$$

# Why two hidden label fields ?



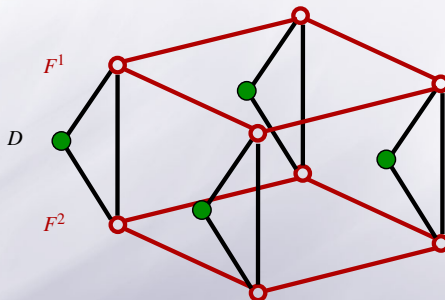
input



segmentation

- ≡ The prior should regularize fields which directly correspond to the natural process “creating” the contents.
- ≡ A correct estimation of the covered verso pixels, through the spatial interactions encoded in the MRF, helps to correctly estimate verso pixels which are **not** covered by a recto pixel.

# The double hidden Markov random field



Observed  
variable



Hidden  
variable



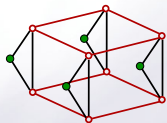
Clique of the  
Prior model



Clique of the  
observ. model



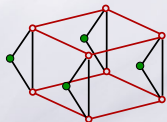
# The joint probability : dependence of recto and verso



- ≡ w/o observations,  $f^1$  and  $f^2$  are independent
- ≡ with observations, they are dependent

$$\begin{aligned}
 & P(f^1, f^2, d) \\
 &= \frac{1}{Z} \exp \{ - (U(f^1) + U(f^2) + U(f^1, f^2, d)) / T \} \\
 &= \frac{1}{Z_1} \exp \{ -U(f^1, f^2) / T \} \cdot \\
 &\quad \frac{1}{Z_2} \exp \{ -U(f^1, f^2, d) / T \} \\
 &= P(f^1, f^2) P(d | f^1, f^2) \\
 &= P(f^1) P(f^2) P(d | f^1, f^2)
 \end{aligned}$$

# The prior of a single field : the Potts model



$$U(f) = \sum_{\{s\} \in \mathcal{C}_1} \alpha f_s + \sum_{\{s,s'\} \in \mathcal{C}_2} \beta_{s,s'} \delta_{f_s, f_{s'}}$$

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases}$$

≡ Discontinuity preserving

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# The observation model (Likelihood)

## Assumption

Two independent degradation processes for each side, followed by a combination of the two “virtual” observations.

$$D = \phi_c(\phi_1(F^1), \phi_2(F^2))$$

The combination process  $\phi_c$  assumes **100% opaque ink** : a recto pixel  $f_s^1 = 1$  completely covers the verso pixel  $f_s^2$ , whatever its label.

# A Gaussian observation model

$$P(d|f^1, f^2) = \prod_s \mathcal{N}(d_s; \boldsymbol{\mu}_s, \boldsymbol{\Sigma}_s)$$

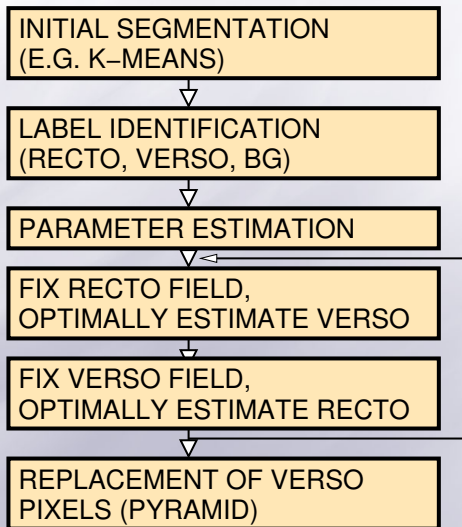
$$\boldsymbol{\mu}_s = \begin{cases} \boldsymbol{\mu}_r & \text{if } f_s^1 = \textit{text} \\ \boldsymbol{\mu}_v & \text{if } f_s^1 = \textit{background} \text{ and } f_s^2 = \textit{text} \\ \boldsymbol{\mu}_{bg} & \text{else} \end{cases}$$

$$\boldsymbol{\Sigma}_s = \begin{cases} \boldsymbol{\Sigma}_r & \text{if } f_s^1 = \textit{text} \\ \boldsymbol{\Sigma}_v & \text{if } f_s^1 = \textit{background} \text{ and } f_s^2 = \textit{text} \\ \boldsymbol{\Sigma}_{bg} & \text{else} \end{cases}$$

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# Outline of the whole method



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# The posterior probability and its maximization



$P(d)$  does not depend on  $f^1, f^2$

$\implies$

We can maximize the joint probability

instead of the posterior probability

$$P(f^1, f^2 | d) = \frac{1}{P(d)} P(f^1, f^2) P(d | f^1, f^2)$$

$$\propto P(f^1, f^2) P(d | f^1, f^2)$$

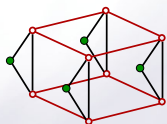
$$U(f^1, f^2, d)$$

$$= \sum_{\{s\} \in \mathcal{C}_1} \alpha^1 f_s^1 + \sum_{\{s, s'\} \in \mathcal{C}_2} \beta_{s, s'}^1 \delta_{f_s^1, f_{s'}^1}$$

$$+ \sum_{\{s\} \in \mathcal{C}_1} \alpha^2 f_s^2 + \sum_{\{s, s'\} \in \mathcal{C}_2} \beta_{s, s'}^2 \delta_{f_s^2, f_{s'}^2}$$

$$+ \sum_{\{s\} \in \mathcal{C}_1} \frac{1}{2} (d_s - \mu_s)^T \Sigma_s^{-1} (d_s - \mu_s)$$

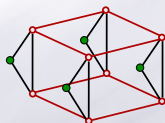
# The posterior probability and its maximization



Rewrite of the energy in terms of unary functions  $U_1$  and two types of binary functions  $U_2$  and  $U'_2$

$$\begin{aligned}
 & U(f^1, f^2, d) \\
 &= \sum_{\{s\} \in \mathcal{C}_1} [\alpha^1 U_1(f_s^1) + \alpha^2 U_1(f_s^2)] \\
 &+ \sum_{\{s, s'\} \in \mathcal{C}_2} [\beta_{s, s'}^1 U_2(f_s^1, f_{s'}^1) + \beta_{s, s'}^2 U_2(f_s^2, f_{s'}^2)] \\
 &+ \sum_{\{s\} \in \mathcal{C}_1} U'_2(f_s^1, f_s^2; d_s)
 \end{aligned}$$

# Minimization with minimum cut/maximum flow



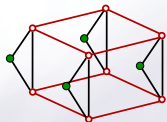
[Kolmogorov et al., PAMI 2004] :

$E$  is *graph-representable*  $\iff$  each binary term  $E(x, y)$  is **submodular** :

$$E(0, 0) + E(1, 1) \leq E(0, 1) + E(1, 0)$$

Depending on the observation  $d_s$  of site  $s$  this may or may not be the case for the observation model.

# Minimization with minimum cut/maximum flow

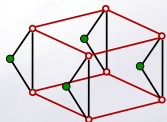


Iterative optimization algorithm :

## Fix/estimate

- ≡ Initialize labels, e.g. with k-means
- ≡ repeat
  - Fix  $f^1$ , estimate optimal  $f^2$
  - Fix  $f^2$ , estimate optimal  $f^1$
- ≡ until happy/dead/running out of patience
  
- ≡ Sub problem can be solved exactly with minimum cut/maximum flow
- ≡ Equivalent to the  $\alpha$ -expansion move algorithm for a single MRF with adapted interaction potentials.

# Minimization with minimum cut/maximum flow

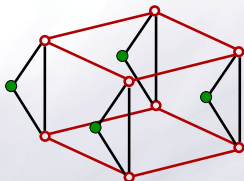


## Joint estimation of a subset of the graph $G$

- ≡ Determine **submodular** sites :  
 $H_s = 1 \iff U'_2(f_s^1, f_s^2, d_s)$  is submodular
- ≡ Initialize labels, e.g. with k-means
- ≡ repeat
  - Fix  $f_s^1$  for all  $s$  and  $f_s^2$  for  $H_s = 0$ , estimate optimal  $f_s^2$  for all  $s$  and  $f_s^2$  for  $H_s = 1$
  - Fix  $f_s^2$  for all  $s$  and  $f_s^1$  for  $H_s = 0$ , estimate optimal  $f_s^1$  for all  $s$  and  $f_s^1$  for  $H_s = 1$
- ≡ until happy/dead/running out of patience

$\implies$  larger standard move !

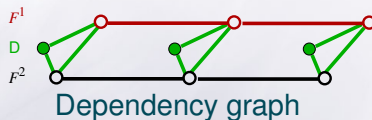
# Minimization with minimum cut/maximum flow



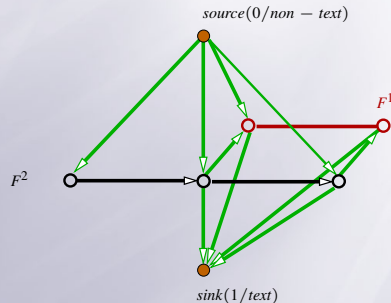
- ≡ Iterative algorithm : one field is fixed (e.g.  $f^1$ , the other field is estimated (e.g.  $f^2$ ).
- ≡ Submodular sites ( $H_s \leftarrow 1$ ) are jointly estimated.

$$\begin{aligned}
 U(f^1, f^2, d) &= \\
 &= \sum_{\{s\} \in \mathcal{C}_1: H_s=0} \alpha^1 U_1(f_s^1) \\
 &+ \sum_{\{s\} \in \mathcal{C}_1: H_s=1} \alpha^1 U_1(f_s^1) \\
 &+ \sum_{\{s\} \in \mathcal{C}_1} \alpha^2 U_1(f_s^2) \\
 &+ \sum_{\{s, s'\} \in \mathcal{C}_2: H_s=0 \wedge H_{s'}=0} \beta_{s, s'}^1 U_2(f_s^1, f_{s'}^1) \\
 &+ \sum_{\{s, s'\} \in \mathcal{C}_2: H_s=1 \wedge H_{s'}=1} \beta_{s, s'}^1 U_2(f_s^1, f_{s'}^1) \\
 &+ \sum_{\{s, s'\} \in \mathcal{C}_2: H_s \neq H_{s'}} \beta_{s, s'}^1 U_2(f_s^1, f_{s'}^1) \\
 &+ \sum_{\{s, s'\} \in \mathcal{C}_2} \beta_{s, s'}^2 U_2(f_s^2, f_{s'}^2) \\
 &+ \sum_{\{s\} \in \mathcal{C}_1: H_s=0} U'_2(f_s^1, f_s^2; d_s) \\
 &+ \sum_{\{s\} \in \mathcal{C}_1: H_s=1} U'_2(f_s^1, f_s^2; d_s)
 \end{aligned}$$

# The cut graph



Cut graph  $\alpha$ -exp move

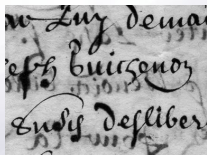


Cut graph proposed algorithm

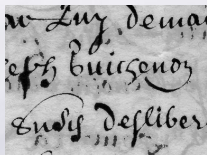
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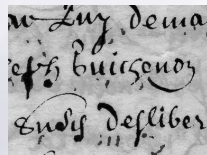
# Examples



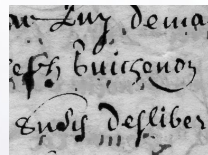
Input



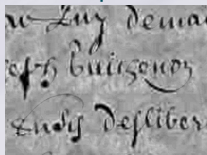
K-means



Single MRF



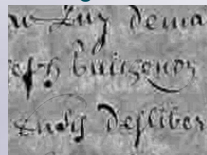
Double MRF



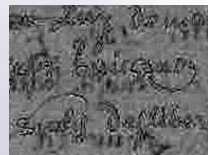
Tonazzini et al.  
(2007) src #1



Tonazzini et al.  
(2007) src #2



Tonazzini et al.  
(2004) src #1



Tonazzini et al.  
(2004) src #2

# Experiments : synthetic & empirical

## Synthetic images

Noiselevel	$\sigma=10$	$\sigma=15$	$\sigma=20$
K-Means	0.25	1.40	3.56
Single-MRF	0.03	0.23	0.73
Double-MRF	<b>0.01</b>	<b>0.08</b>	<b>0.31</b>

## Empirical evaluation (16 people $\times$ 4 test images)

Method	complete			no single-MRF	
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	1 <sup>st</sup>	2 <sup>nd</sup>
K-Means	18	10	36	21	43
Single-MRF	13	39	12	-	-
Double-MRF	<b>33</b>	15	16	<b>43</b>	21
Total	64	64	64	64	64

# OCR improvement

Method	Recall	Prec.	Cost	Dataset
No restoration	65.65	49.91	76,752	100
Niblack (segm. only)	-	-	-	-
Sauvola et al. (segm. only)	78.75	66.78	45,363	100
K-Means (k=3)	78.57	69.43	40,375	100
Tonazzini et al. meth. 1 - src #1	41.00	30.05	74,819	66
Tonazzini et al. meth. 1 - src #2	-	-	-	-
Tonazzini et al. meth. 2 - src #1	-	-	-	-
Tonazzini et al. meth. 2 - src #2	-	-	-	-
Tonazzini et al. meth. 2 - 3 src	50.52	33.90	101,280	89
Single MRF & $\alpha$ -exp. move	81.99	72.12	36,744	100
Double MRF (proposed method)	<b>83.23</b>	<b>74.85</b>	<b>32,537</b>	100

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Input

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k-means

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single MRF

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double MRF

# Conclusion and Outlook

- ≡ The double MRF model is more natural and more powerful than the single MRF model.
- ≡ The double MRF model using binary labels naturally leads to an efficient graph cut implementation.
- ≡ The MRF methods based on classification beat the source separation based methods in terms of OCR improvement.

## Journal paper

More details in

C. Wolf, *Document Ink bleed-through removal with two hidden Markov random fields and a single observation field*, to appear in IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI)