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Generalizations of angular radial transform for 2D and 3D shape retrieval

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Abstract

The angular radial transform (ART) is a moment-based image description method adopted in MPEG-7 as a 2D region-based shape descriptor. This paper proposes generalizations of the ART to describe two-dimensional images and three-dimensional models. First, we propose an 2D extension, called GART, which allows applying ART to images while insuring robustness to all possible rotations and to perspective deformations. Then, we generalize the ART to index 3D models. This new 3D shape descriptor, so called 3D ART, has the same properties that the original transform: robustness to rotation, translation, noise and scaling while keeping a compact size and a good retrieval cost. The size of the descriptor is an essential evaluation parameter on which depends the response time of a content-based retrieval system. For both generalizations, many experiments were made on large databases and have shown, that GART outperforms ART in accuracy at the cost of speed, and that 3D ART outperforms the spherical harmonics shape descriptor (Vranic, D.V., Saupe, D., 2002. Description of 3D-shape using a complex function on the sphere, in: IEEE International Conference on Multimedia and Expo (ICME 2002), Lausanne, Switzerland, 2002, pp. 177–180; Funkhouser, T., Min, P. Kazhdan, M., Chen, J., Halderman, A., Dobkin, D., Jacobs, D., 2003. A search engine for 3D models. ACM Trans. Graphics 22(1), 83–105) in speed at the cost of accuracy.

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1. Introduction

Content-based image retrieval has been a topic of intensive research in recent years, and particu-

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larly the development of effective shape descriptors (SD). The MPEG-7 standard committee has proposed a region base shape descriptor, the angular radial transform (ART) (Jeannin, 2001; Kim and Kim, 1999). This SD has many properties: compact size, robustness to noise and scaling, invariance to rotation, ability to describe complex objects. These properties and the evaluation made during the MPEG-7 standardization process make the ART a unanimously recognized efficient descriptor. Furthermore, an important characteristic is the small size of the ART descriptor. For a huge database, this implies fast answers during retrieval processes. In the MPEG-7 standard, the ART similarity measure is reduced to a L_1 distance between 35 floating point values.

In the same time, the technical 3D model databases grow up since the beginning of the computer-aided design. The engineering laboratories and the design offices always increase the number of 3D solid objects and the current industrial estimations point to the existence of over 30 billion of CAD models.

This huge number of models requires a content-based mining with indexing and retrieval processes. In the framework of the Semantic-3D national project and in partnership with the car manufacturer Renault, we investigate the possibilities to make a fast descriptor to index a huge technical 3D models database and to index images to insure robustness to deformation undergone by objects in natural images. In this context, we explore the possibilities to extend ART to the retrieval of images and 3D models by taking into account the specific properties of these data.

This article presents our work on the Angular Radial Transform. First, we generalize the 2D ART shape descriptor to insure robustness to perspective deformations that can disturb a planar shape in a 2D natural image. In a second time, the ART is extended for the indexation of 3D models while preserving its properties. This paper is organized as follows: Section 2 presents the ART transform, Section 3 details the generalization of the 2D ART, Section 4 presents a survey of the related work on 3D shape matching and our new 3D ART descriptor, results are presented and discussed in the last section.

2. The angular radial transform

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This part presents the 2D ART proposed by the MPEG-7 normalization process. These definitions are the starting point of the proposed generalizations. Angular radial transform (ART) is a moment-based image description method adopted in MPEG-7 as a region-based shape descriptor (Bober, 2001). It gives a compact and efficient way to express pixel distribution within a 2D object region; it can describe both connected and disconnected region shapes. The ART is a complex orthogonal unitary transform defined on a unit disk based on complex orthogonal sinusoidal basis functions in polar coordinates (Jeannin, 2001; Kim and Kim, 1999). The ART coefficients, F_{nm} of order n and m , are defined by

$$F_{nm} = \int_0^{2\pi} \int_0^1 V_{nm}(\rho, \theta) f(\rho, \theta) \rho d\rho d\theta \quad (1)$$

where $f(\rho, \theta)$ is an image function in polar coordinates and $v_{nm}(\rho, \theta)$ is the ART basis function that is separable along the angular and radial directions:

$$V_{nm}(\rho, \theta) = A_m(\theta)R_n(\rho)$$

$$\text{where } \begin{cases} A_m(\theta) = \frac{1}{2\pi} \exp(jm\theta) \\ R_n(\rho) = \begin{cases} 1 & n = 0 \\ 2 \cos(\pi n \rho) & n \neq 0 \end{cases} \end{cases} \quad (2)$$

In order to achieve rotation invariance, an exponential function is used for the angular basis function. The radial basis function is defined by a cosine function. Real parts of basis functions are shown in Fig. 1.

The ART descriptor is defined as a set of normalized magnitudes of the set of ART coefficients. Rotational invariance is obtained by using the magnitude of the coefficients. In MPEG-7, 12 angular and three radial functions are used ($n < 3$, $m < 12$) (Jeannin, 2001), these values will be used in the rest of the articles. For scale normalization, the ART coefficients are divided by the magnitude of the ART coefficient of order $n = 0$, $m = 0$. The distance between two shapes described by the ART descriptor is calculated using L_1 norm:

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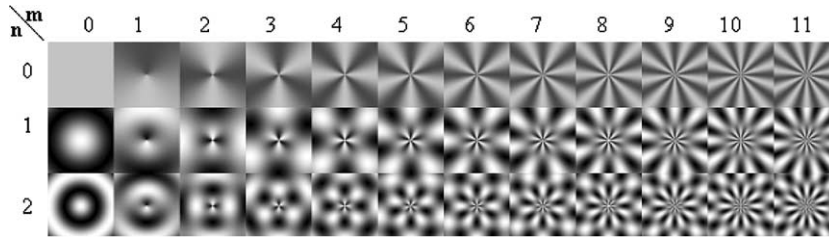


Fig. 1. Real parts of the ART basis functions.

$$d_{\text{ART}}(Q, I) = \sum_{i=0}^{n-m} \|\text{ART}_Q[i] - \text{ART}_I[i]\| \quad (3)$$

122 The subscripts Q and I represent respectively the
 123 query image and an image in the database, and
 124 ART_I is the array of the normalized ART coefficients
 125 of the image I . Note that to decrease the
 126 descriptor size, quantification can be applied to
 127 each coefficient using four bits per coefficient
 128 (Jeannin, 2001). The MPEG-7 standardization
 129 process showed the efficiency of the method in
 130 the 2D indexing field. We can quote the use of
 131 ART in a multi-views 3D models retrieval (Chen
 132 and Ouhyoung, 2002), and in face detection (Fang
 133 and Qiu, 2003).

134 To use ART on a natural color image and to
 135 take into account the internal variations of the ob-
 136 jects (contours, holes, texture,...), the ART
 137 descriptor can be computed on the luminance
 138 component. In that case the function $f(\rho, \theta)$ takes
 139 the values in the interval $[0, 1]$ (see Wang et al.,
 140 xxxx; Laaksonen et al., xxxx; Akcay et al., 2002).

3. Generalization of ART to perspective deformations

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143 The goal of this generalization is to make the
 144 ART robust to any rotations or perspective projec-
 145 tions. A planar object in a natural scene can be
 146 viewed according to all orientations and can be
 147 carried by an unspecified plan. This highly proba-
 148 ble situation will disturb the shape in the image
 149 and will prevent the identification. In Fig. 2, a
 150 plane object (a stamp) is seen with three angles
 151 of acquisition which correspond to three different
 152 shapes projected on the same image plane. To
 153 make ART descriptor robust to all possible rota-
 154 tions and to perspective projections it is necessary
 155 to generalize the ART transform with new basis
 156 functions. This new descriptor is called generalized
 157 ART (GART).

158 In order to define the transformations under-
 159 gone by an object during rotations and projections
 160 onto the image plane, we consider the transforma-
 161 tion space given by the perspective coefficient p
 162 and the normal vector to the image plane, denoted
 163 \vec{n} . The perspective coefficient p defines a distance

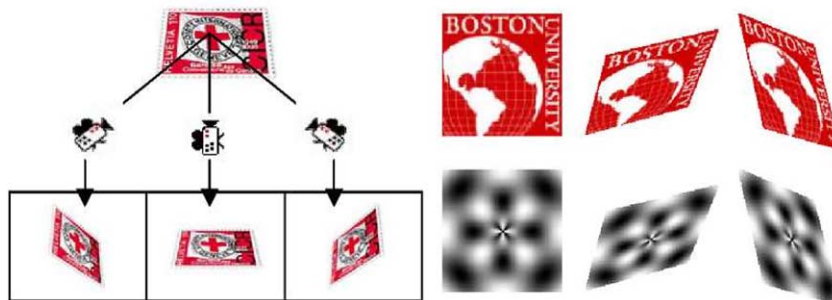


Fig. 2. Object seen according to various angles and example of projected basic functions on the object support plane.

164 between the original basis function and the image
 165 plane. The normal vector \vec{n} is given by its Euler angles ζ (radial direction) and ϕ (rotation angle) defined
 166 between the vectors \vec{n} and the axis \vec{x} (see the
 167 Fig. 3). The first two parameters define the orientation
 168 of the normal to the image plane and the
 169 third is the perspective coefficient which defines
 170 the perspective deformation. Fig. 3 shows these
 171 parameters.

173 This transformation space is sampled for each
 174 parameter according to k_ζ , k_ϕ and k_p values.
 175 Hence we obtain a sampling of $K = k_\zeta * k_\phi * k_p$
 176 transformations. The basis functions are deformed
 177 in the same way according to the K transformations.
 178 Each object is indexed with these K sets of
 179 projected basis functions. The number of projections
 180 is limited to keep a reasonable computational
 181 cost. The values $k_\zeta = 12$, $k_\phi = 3$ and $k_p = 3$, are
 182 chosen in our experiments presented in Section 5,
 183 because these values give the better ratio of cost
 184 to efficiency. In other words, we have $K = 108$ sets
 185 of coefficients to describe a shape. Hence we have
 186 to compute 108 similarity measures between a
 187 query object and a database object.

188 The complexity of the classical ART is in
 189 $\theta(n * m * N^2)$ because we compute $n * m$ basis
 190 functions values for the $N * N$ pixels of the image.
 191 The generalized ART, which creates K set of basis
 192 functions, has a complexity in $\theta(K * n * m * N^2)$.

193 To make the retrieval process faster, we choose
 194 to inverse the indexation and retrieval processes.
 195 Without optimization, the indexing process computes
 196 the ART descriptor between the original object
 197 and the original basis functions whereas the
 198 retrieval process computes the descriptor between
 199 the extracted object from a natural image and all
 200 the projected basis functions. Thus, the indexation

201 process, which is offline, has a computation cost K
 202 times less than the retrieval process, which is on-
 203 line. Fortunately, it is possible to inverse these
 204 two processes and to index the original objects
 205 on the inverse projected basis functions, whereas
 206 an extracted object will be indexed only on the original
 207 basis functions. This increases the cost of the
 208 offline indexing process but decreases the online retrieval
 209 process without modification of the description
 210 (Table 1).

211 As it is shown in Fig. 4, we transform the original
 212 image F_0 to the deformed image F_1 by a transformation
 213 T and we transform the origin basis
 214 function V_0 to the inverse projected basis function
 215 V_{-1} by a transformation T^{-1} . We can see that we
 216 obtained the same descriptor values, if we index
 217 the deformed image F_1 on the original basis function
 218 V_0 , or if we index the origin image F_0 on the
 219 inverse basis function V_{-1} .

220 Each object is described by $K = k_\zeta * k_\phi * k_p$ series
 221 of ART coefficients created from the basis functions
 222 projected on K planes of projections. The shape
 223 similarity distance is achieved by computing
 224 a set of distances $d_{\text{ART}}(Q, I_j)$. For each value of j ,
 225 the ART coefficients of Q , computed on the original
 226 basis functions, and those of I , computed on the
 227 j th projection of the basis functions, are compared
 228 using (3). Then the shape distance between
 229 Q and I is given by

Table 1
 Number of online and offline ART descriptor computation
 during the original process and the optimized one

	Original process	Optimized process
Online	K	1
Offline	1	K

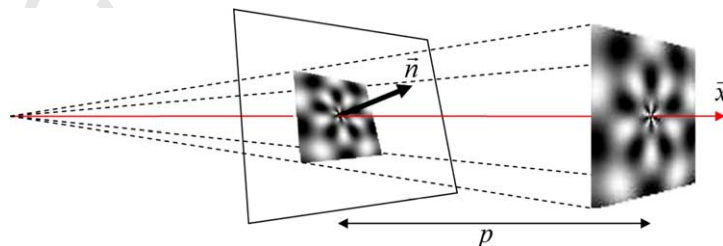


Fig. 3. The basis functions are projected on the image plane I according to ζ , ϕ and p to obtain the projected basis functions.

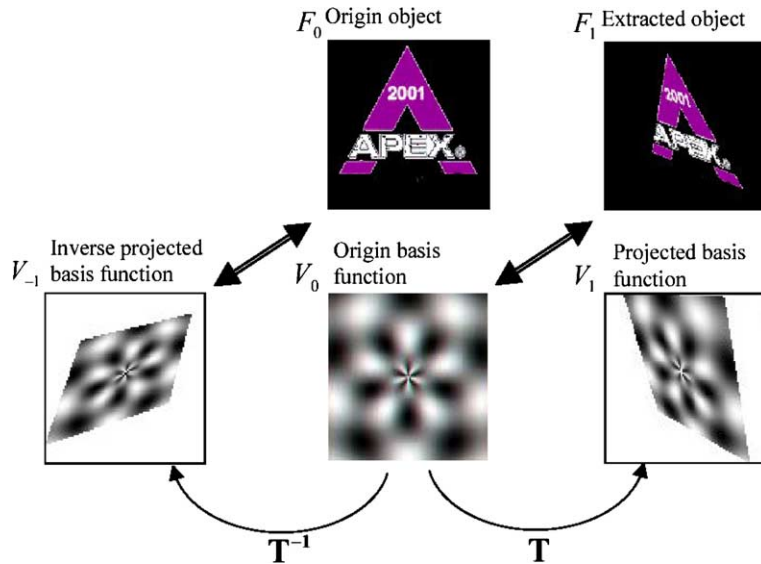


Fig. 4. Diagram of inverse indexation process.

$$d_{\text{shape}}(Q, I) = \min_{j \in K} \sum_{i=0}^{n-m} \|\text{ART}_Q[i] - \text{ART}_I^j[i]\| \quad (4)$$

232 where Q is the ART coefficients of the key object
 233 and I_j is the coefficients of the I object, calculated
 234 on the j th projection of the basis functions. The
 235 minimum is considered in order to take into ac-
 236 count all the possible perspective views of the
 237 object.

238 4. 3D angular radial transform

239 In this section, we present a survey of the re-
 240 lated works on 3D shape matching, then we gener-
 241 alize the MPEG-7's angular radial transform to
 242 the 3D space.

243 4.1. Survey of recent 3D indexing methods

244 3D indexing methods can be divided into two
 245 distinct groups: retrieval by an example of a
 246 three-dimensional model, and retrieval by a 2D
 247 view. In this work, we are interested in 3D model
 248 retrieval. The state of the art can be divided into
 249 two different classes of 3D shape description meth-

ods: the structural approaches and the statistical
 approaches.

250 The structural approach is a high-level one
 251 which aims to describe the shape in a more com-
 252 plete and intuitive manner. The principle is to split
 253 an object into sub-parts and to represent the object
 254 as the merge of these sub-parts according to adja-
 255 cency relationships. A segmentation step identifies
 256 the elementary structures composing the objects
 257 satisfying given homogeneity criteria. The deter-
 258 mined components are represented by using some
 259 specific structures such as trees or graphs. Two dis-
 260 tinct approaches can be considered: the surface-
 261 based approaches and the volume-based ap-
 262 proaches. A surface-based approach segments sur-
 263 faces into patches. The connectivity of such
 264 patches is encoded within an adjacency graph. A
 265 similarity measure is computed between two ob-
 266 jects by graph matching techniques (Ullmann,
 267 1976). Dorai and Jain (1997) proposes to use a
 268 graph of maximal patches defined by functions
 269 of the principal curvature. On 3D technical mod-
 270 els, the model signature graph (MSG) (McWherter
 271 et al., 2001) is constructed by a surface-based rep-
 272 resentation of the object. Each face is represented
 273 by valued vertices and valued edges exist if two
 274 vertices are adjacent. Hilaga et al. (2001) uses mul-
 275 tiple

277 ti-resolution Reeb graph. Multi-resolution graph
 278 are constructed by computing a surface geodesic
 279 distance to define a Reed graph at various levels.
 280 Recently, the Augmented Reeb Graphs (Tung
 281 and Schmitt, accepted for publication) increases
 282 the matching process. The volume-based ap-
 283 proaches decompose a shape using 3D elementary
 284 volumetric structures called geons (geometrics
 285 ions) based on recognition by composant theory
 286 (Biederman, 1987). The sets of 3D volumetric
 287 primitives may be: cylinders, cubes, parallelepi-
 288 peds, cone (truncated or not), ellipsoids (Irani
 289 and Ware, 2000). Other interesting approaches
 290 use a set of superquadrics (Zhou and Kambha-
 291 mettu, 2002) or quadratic surfaces (Park et al.,
 292 2002).

293 The statistical approaches characterize the 3D
 294 model shape by calculating statistical moments
 295 (Zhang and Chen, 2001) or by considering a distri-
 296 bution of the measurement of geometric primitives
 297 (which might be points, cords, triangles, tetrahe-
 298 drons,...) (Osada et al., 2001). A geometrical nor-
 299 malization of the object size and position in a 3D
 300 space is used as a pre-processing step to guarantee
 301 a geometric invariance. The moment-based ap-
 302 proaches can be defined as projections of the func-
 303 tion defining the object onto a set of characteristic
 304 moment functions. These approaches are used in
 305 2D pattern recognition with several 2D moments:
 306 geometrical, Legendre, Fourier–Mellin, Zernike,
 307 pseudo-Zernike moments (Teh and Chin, 1988)
 308 and ART (Jeannin, 2001; Kim and Kim, 1999).
 309 Some of these moments have been extended into
 310 3D: 3D Fourier (Elad et al., 2001), 3D Wavelet
 311 (Paquet and Rioux, 2000), 3D Zernike (Cantera-
 312 kis, 1999) and the spherical harmonic (SH) decom-
 313 position, recently described by Vranic and Saupe
 314 (2002), and Funkhouser et al. (2003). The spherical
 315 harmonic analysis decomposes a 3D shape into
 316 irreducible sets of rotation independent compo-
 317 nents by sampling the three-dimensional space

318 with concentric shells, where the shells are defined
 319 by equal radial intervals. The spherical functions
 320 are decomposed as a sum of the first 16 harmonic
 321 components (Kazhdan et al., xxxx), in an analog-
 322 ous way to the Fourier decomposition into differ-
 323 ent frequencies. Using the fact that rotations do
 324 not change the norm of the harmonic components,
 325 the signature of each spherical function is defined
 326 as a list of these 16 norms. Finally, these different
 327 signatures are combined to obtain a $32 * 16$ signa-
 328 ture vector for each 3D model. During the retrie-
 329 val step, the similarity of objects is calculated as
 330 the Euclidean distance between these vectors. In
 331 our experimentation, the proposed descriptor 3D
 332 ART is compared to SH.

4.2. 3D ART definition

334 First, we suppose the objects to be represented
 335 in spherical coordinates where ϕ is the azimuthal
 336 angle in the xy -plane from the x -axis, θ is the polar
 337 angle from the z -axis and ρ is the radius from a
 338 point to the origin. The 3D ART is a complex uni-
 339 tary transform defined on a unit sphere. The 3D
 340 ART coefficients are defined by

$$F_{nm_\theta m_\phi} = \int_0^{2\pi} \int_0^\pi \int_0^1 V_{nm_\theta m_\phi}(\rho, \theta, \phi) \times f(\rho, \theta, \phi) \rho d\rho d\theta d\phi \quad (5)$$

343 where $F_{nm_\theta m_\phi}$ is the 3D ART coefficient of orders n ,
 344 m_θ and m_ϕ , $f(\rho, \theta, \phi)$ is a 3D object function in
 345 spherical coordinates and $V_{nm_\theta m_\phi}(\rho, \theta, \phi)$ is a 3D
 346 ART basis function (BF). The 3D BFs are separa-
 347 ble along the angular and the two radial directions:

$$V_{nm_\theta m_\phi}(\rho, \theta, \phi) = A_{m_\theta}(\theta)A_{m_\phi}(\phi)R_n(\rho) \quad (6)$$

350 The radial basis function is defined by a cosine
 351 function and the angular basis functions are def-
 352 ined by complex exponential functions to achieve
 353 rotation invariance and continuity along both θ
 354 and ϕ values:

$$R_n(\rho) = \begin{cases} 1 & n = 0 \\ 2\cos(\pi n\rho) & n \neq 0 \end{cases} \quad \text{and} \quad \begin{cases} A_{m_\theta}(\theta) = \frac{1}{2\pi} \exp(2jm_\theta\theta) \\ A_{m_\phi}(\phi) = \frac{1}{2\pi} \exp(jm_\phi\phi) \end{cases} \quad (7)$$

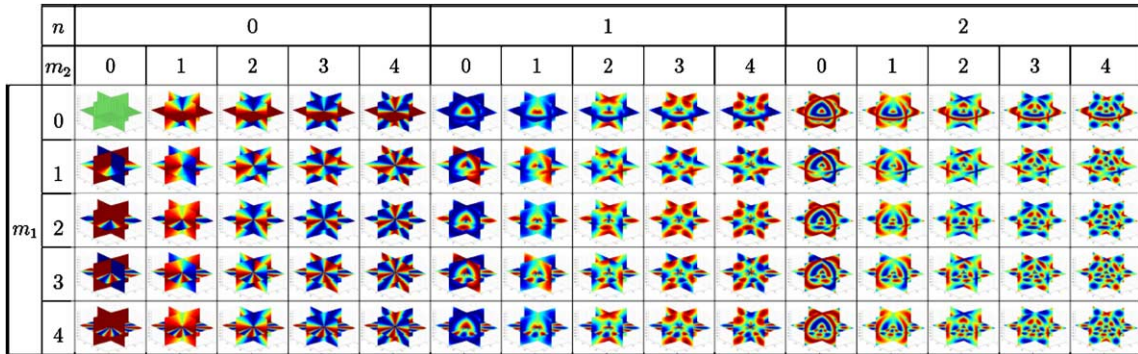


Fig. 5. Real parts of 3D ART BF.

355 The values of the parameters n , m_θ and m_ϕ are
 356 trade-offs between efficiency and accuracy. Choices
 357 were made by computing the Recall response for
 358 different sets of values. For the technical database
 359 presented in Section 4, we have chosen $n = 3$,
 360 $m_\theta = 5$ and $m_\phi = 5$. The real parts of the 3D
 361 ART BFs are shown in Fig. 5.

362 The similarity measure is computed using a L_1
 363 distance between the 3D ART descriptors:

$$d(Q, I) = \sum_{i=1}^{n \cdot m_\theta \cdot m_\phi} \|\text{ART3D}_Q[i] - \text{ART3D}_I[i]\| \quad (8)$$

366 where Q and I represent respectively a query object
 367 and an object of the database and ART3D is the
 368 array of 3D ART descriptor values normalized
 369 by F_{000} . The choice of the L_1 distance is justified
 370 by speed preoccupations but other distances could
 371 be used.

372 4.3. Indexing process

373 An important property of the 2D ART is the
 374 rotation invariance. A 2D rotation representation
 375 in polar coordinates can be express as the sum of
 376 angular components:

$$(\rho, \phi) \xrightarrow{\text{Rot}_z} (\rho, \phi + \alpha) \quad (9)$$

379 That does not modify the norm of the function
 380 $A_{m_\theta}(\theta)$ and therefore nor the ART descriptor. In
 381 3D, unspecified rotations cannot be expressed as
 382 the sum of constant values on the angular compo-
 383 nents, and thus modify the descriptor values.
 384 However, if we consider a rotation around the z -

axis, the norms of the 3D ART coefficients do
 385 not change. Hence, to have a rotation invariance,
 386 unspecified rotations must be transformed to rota-
 387 tions along z -axis by alignment according to the
 388 first principal direction. Thus, a principal compo-
 389 nents analysis (PCA) is applied to obtain the prin-
 390 cipal direction of the objects. PCA alignment is
 391 not really robust when the three principal direc-
 392 tions are considered. Fortunately, here, we only
 393 need to align the first principal direction along
 394 the z -axis, therefore wrong alignments are limited.
 395 Fig. 6 shows the indexation process.
 396

Hence, before projecting 3D models onto the
 397 BFs, the objects are pre-processed as follows: first,
 398 they are discretized in a grid in such a way to ob-
 399 tain interior and exterior voxels. This discretiza-
 400 tion is also used to compute the parameters of
 401 centering, scaling and alignment to the z -axis: the
 402 3D object is centered on its gravity center and
 403 scaled up. This pre-processing step makes the 3D
 404 ART robust to translations, rotations and scaling.
 405 Finally, the discretized object is projected into the
 406 3D ART BFs to obtain the 3D ART coefficients.
 407

5. Experiments

This part shows the experiments that we have
 409 made to evaluate the ART generalizations. First,
 410 we present our tests on the 2D GART, then we
 411 present the 3D model databases and the experi-
 412 ments that we have made to illustrate the proper-
 413 ties and the effectiveness of the 3D ART.
 414

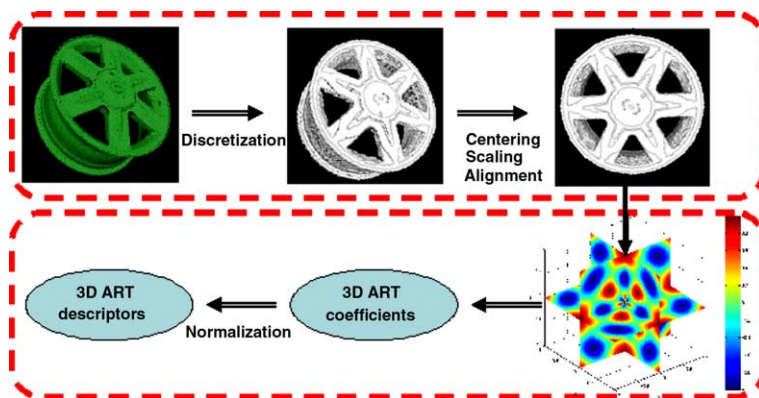


Fig. 6. Indexing process.

415 5.1. 2D generalized ART experiments

416 In these experiments, ART and GART was
 417 used on the luminance components of the de-
 418 scribed images like explained in part 2. The first
 419 test compares the ART on the luminance and
 420 our GART extension. 1813 test images were cre-
 421 ated from 37 trademark images disturbed accord-
 422 ing to 49 random perspective projections with
 423 illuminating variations and grouped in 37 classes.
 424 The original trademark images were used as query
 425 images the answer ranks of the class images were
 426 evaluated. Note that the GART descriptor is 108
 427 times larger than the ART descriptors and the simi-
 428 larity measures compute have a same cost differ-
 429 ences. Fig. 7a shows the recall/precision values.
 430 This curve shows that the best results are obtained

with GART. GART is found to be more accurate
 but slower than ART.

The GART was defined for a detection applica-
 tion of a trademark in natural images. This appli-
 cation identifies an object extracted from an
 image. Its general scheme can be seen in Fig. 8
 and can be split into two successive steps: the
 indexation and the retrieval. To evaluate the prop-
 erties of the GART within the retrieval process, 50
 objects were extracted from the images ranks
 where one finds the original trademark were con-
 sidered. For this application, we have also consid-
 ered a color description to take into account color
 properties. The color descriptor is a simple color
 histogram associated with a histogram similarity.
 The GART and the color description were mixed
 into a global similarity function computed as a

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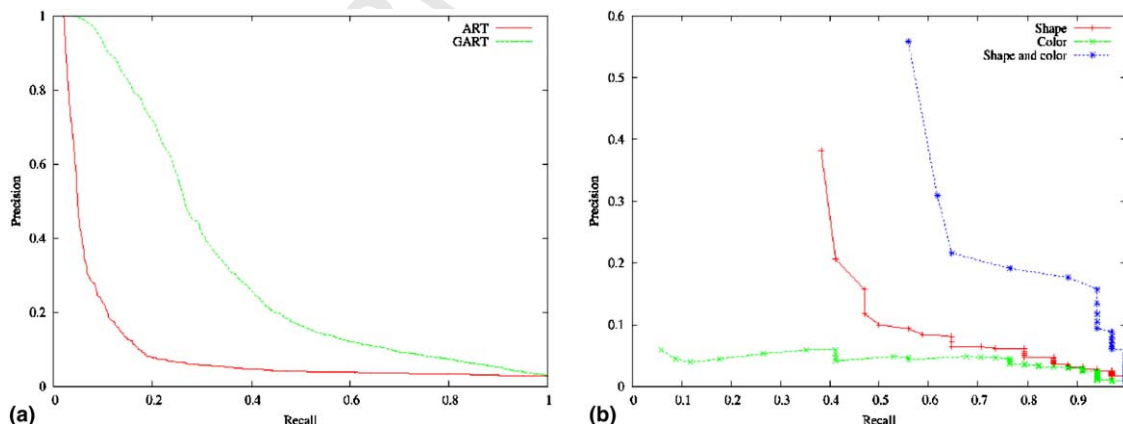


Fig. 7. Recall/precision curves: (a) ART and generalized to perspective projection ART, (b) GART, color and mixed approach.

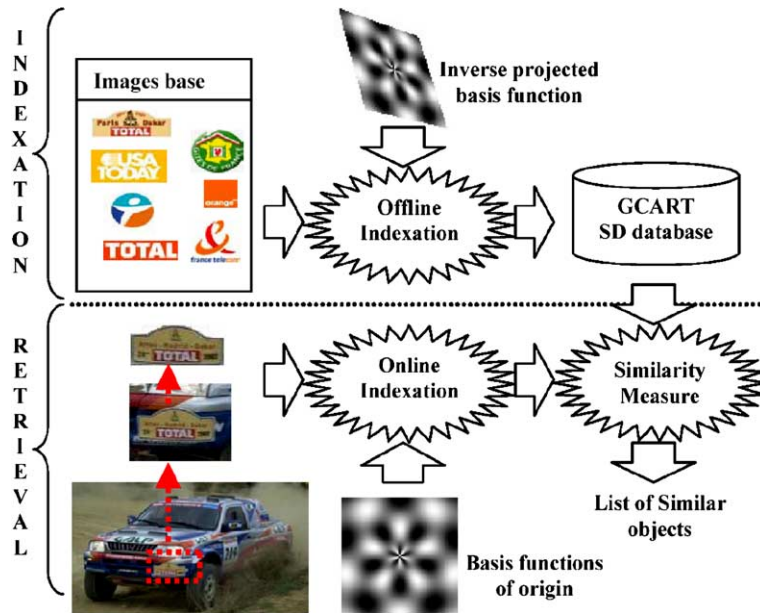


Fig. 8. General diagram of the application.

448 weighted sum of each distances (Idrissi et al.,
 449 2004). The Fig. 7b shows the Recall values for
 450 the GART, the color and the mixed description.
 451 The mixed description gives the original trademark
 452 at the first rank in 55%, against 38% for the GART
 453 and 6% for the color. At the rank 10, the original
 454 trademark is found in 95% of the cases, whereas
 455 the GART and the color study have found the origi-
 456 ninal object, respectively in 65% and 41% of the
 457 cases.

458 5.2. 3D ART experiments

459 5.2.1. 3D model database test

460 The 3D experiments are made using two 3D
 461 model databases: the Princeton Shape Benchmark

(Shilane et al., 2004) and a Renault database. Fig. 462
 463 9 show examples of 3D models both databases.

The Princeton Shape Benchmark provides a 464
 465 repository of 3D models and software tools to
 466 evaluate shape-based retrieval and analysis algo-
 467 rithms. The motivation is to promote the use of
 468 standardized data sets and evaluation methods
 469 for research in matching, classification, clustering,
 470 and recognition of 3D models. The Princeton
 471 database contains 1814 models grouped into
 472 high-level semantic classes where the objects of a
 473 same class are heterogeneous. For example, a class
 474 of staircases contains 3D models, which represent
 475 staircases of very different shape but with the same
 476 semantic (Fig. 10). The Renault database is a tech-
 477 nical database, which contains mechanical models.
 478 In the framework of SEMANTIC 3D and in part-

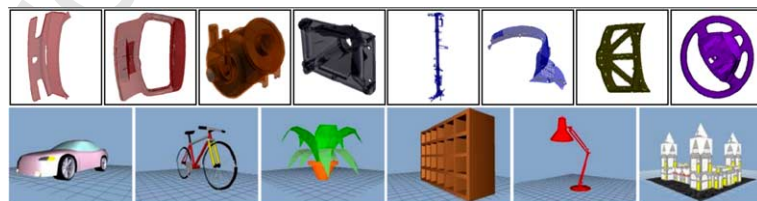


Fig. 9. Examples of Princeton Shape Benchmark 3D models and Renault 3D models.

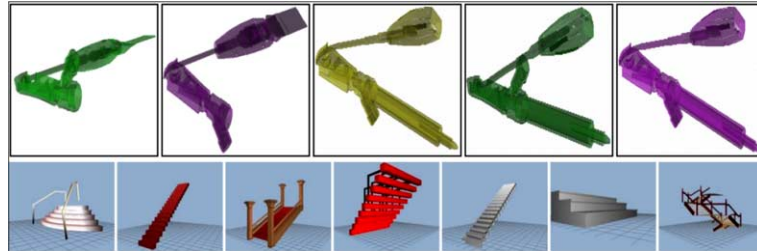


Fig. 10. Example of Princeton Shape Benchmark class: staircase and a Renault database class: seat belt part.

479 nership with the car manufacturer Renault, we
 480 have a huge 3D technical model database (approx-
 481 imately 5000 models). This database contains the
 482 pieces composing a car with all the model versions.
 483 The 5000 models were classified according to the
 484 functionalities of the different parts. 781 objects
 485 were classified in 75 classes. We can quote for
 486 example the classes: wheel, door, brake pad, disc
 487 of brake, bolt,... Not all the database objects
 488 can be classified because the database does not
 489 have enough models to guaranty a minimal num-
 490 ber of models per class. Classes, which have a
 491 number of models less than 5, are grouped in an
 492 unspecified class. The tests were made by taking
 493 all the objects of the specified classes as request ob-
 494 jects for the 5000 object database. The recall and
 495 precision values are the mean of the recall and pre-
 496 cision values of all the objects of the classes. Exam-
 497 ples of the two databases classes are shown in Fig.
 498 10.

5.2.2. ART 3D parameters

499
 500 To fix the parameter values, the recall values are
 501 compared. Twelve values of the parameters n , m_0
 502 and m_ϕ are evaluated. Fig. 11a shows that the best
 503 results are obtained for $n = 3$ and $m_0 = m_\phi = 5$.
 504 Fig. 11b presents the same experiment with differ-
 505 ent discretization sizes S . Better results are ob-
 506 tained on the technical database with the
 507 parameter value $S = 64$. Thus, we use this value
 508 in the rest of this work. This value is also suggested
 509 in (Kazhdan et al., xxxx) for the SH computation.

5.2.3. Robustness

510
 511 To evaluate the robustness of the process, we
 512 distort a 3D object according to scaling, rotation,
 513 translation and noise. Table 2 shows the maximum
 514 and the mean distance obtained for these four dis-
 515 tortions. For each distortion, we create a set of 3D
 516 objects and for all the objects, we compute the dis-
 517 tance to the original one. The translation has no
 518 effect on the distance, because the pre-processing
 519 step centers the objects. For the same reasons,

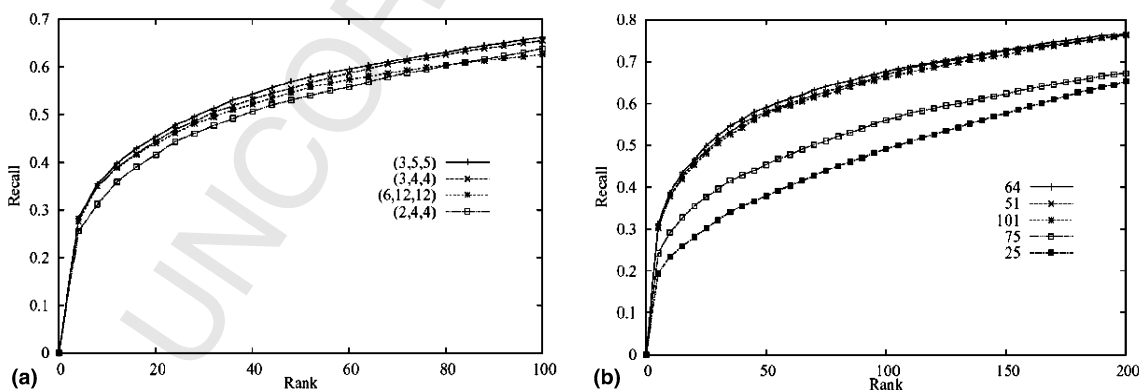


Fig. 11. The Recall values to set up parameters.

Table 2

Distance obtained for several distortions

Distort	Translation	Scale	Rotation	Noise
Max distance	0	0.016	1.272	2.217
Mean distance	0	0.003	0.750	1.012

520 the scale distortion has small effects due to arti-
 521 facts of digitization, the maximum distance be-
 522 tween the scaled objects are 0.016 when a mean
 523 distance between two objects of the same class is
 524 around 3. The obtained distances are smaller than
 525 intra-class distances and the classification is the
 526 same one. The rotation distortion test is a set of
 527 rotations around the three axes with random ang-
 528 gles and gives a maximum distance of 1.272 and
 529 a mean distance of 0.75. The noise distortion is a
 530 random move of vertices of the object; each vertex
 531 is moved along a random Gaussian vector. This
 532 distance is a percentage of the object size. If this
 533 distance is higher than 10% the surface of the ob-
 534 ject is much distorted but the similarity measure is

1.6 and the object are still well classified. Fig. 12 535
 shows distorted objects by the noise distortions. 536

5.2.4. Comparison 537

A second experiment is set up to compare the 538
 3D ART to the Spherical Harmonic descriptor 539
 (SH). This experiment is made on the two model 540
 databases. Fig. 13a and b shows the recall values 541
 for SH and 3D ART descriptors for the two dat- 542
 abases. On the Princeton database (Fig. 13a), the 543
 SH method gives a better description than the 544
 ART. The results on the Renault database are sim- 545
 ilar with the two methods (Fig. 13b). ART descrip- 546
 tion gives better results when the objects of a same 547
 class are similar. The 3D ART goal was not to 548

Table 3

Size (in floating numbers) and indexing time (in seconds)
 comparison between 3D ART and spherical harmonic
 representation

	Indexing time	Descriptor size
SH	10	544
3D ART	4	74

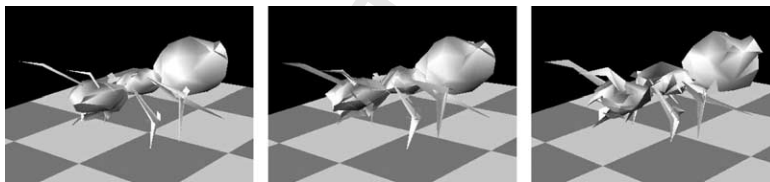


Fig. 12. Example of noise distortions for three distance values: 0%, 5% and 10%.

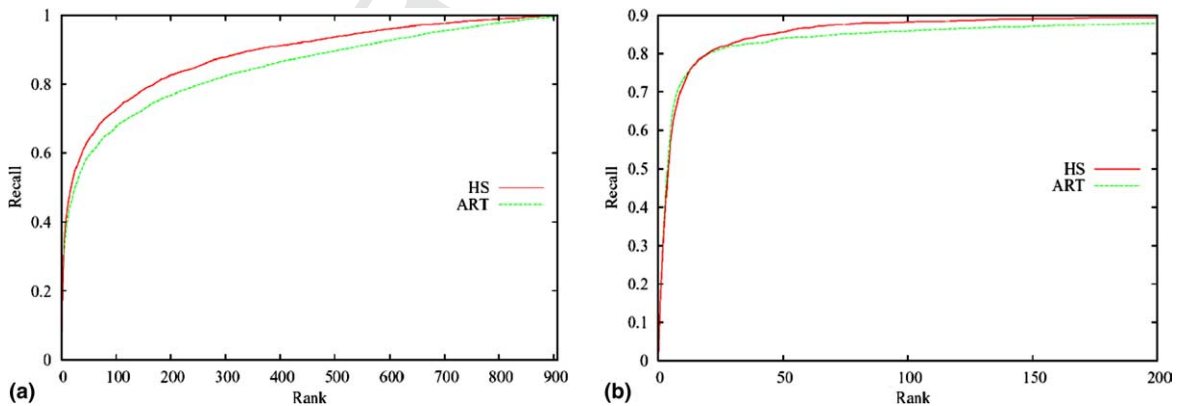


Fig. 13. Recall/precision values on (a) Princeton and (b) Renault databases.

549 have a the best description accuracy, but to make a
550 small descriptor to have a fast answer.

551 The computational cost and the size of the
552 descriptors are significant comparison criteria (Ta-
553 ble 3). The 3D ART indexing computation time is
554 2.5 times less than a SH indexing and the descrip-
555 tor size and the cost of the similarity measure is
556 approximately 7.8 times less. These differences
557 are due to the fact that the ART BF and the inte-
558 gral calculus are defined in the Euclidian space
559 whereas the SH description is computed using
560 complex frequency transformations. In the frame-
561 work of the SEMANTIC 3D project, a huge 3D
562 models database will be index. Thus, the cost of
563 the retrieval must be as small as possible.

564 6. Conclusion

565 In this paper, we have presented an extension of
566 the 2D region-based shape descriptor ART to de-
567 formed images and to 3D models. The generaliza-
568 tion of the ART (GART), to perspective
569 projections, increases the ART efficiency and defi-
570 nition domain while keeping the discriminating
571 capacities. Moreover the optimized process makes
572 possible to have a light online process and a quick
573 answer for content-based image retrieval. We have
574 shown that GART is more accurate than ART at a
575 higher cost.

576 In the second part of this work, we have pre-
577 sented the generalization of the ART to describe
578 3D shape (3D ART). The proposed descriptor is
579 robust to translations, scaling, multi-representa-
580 tion (remeshing, weak distortions), noises and 3D
581 rotations. It fulfils the requirements for our CAD
582 database indexing and retrieval application:
583 robustness and accuracy of the indexing, and
584 high-speed retrieval processes and similarity com-
585 putation index. Moreover experiments have shown
586 that 3D ART outperforms the spherical harmonics
587 descriptor in speed, while keeping a close accuracy.

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(<http://www.semantic-3d.net>). 593

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