Image watermarking technique based on the steerable pyramid transform

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ABSTRACT

The application of the steerable pyramid transform in image watermarking has many useful properties. In this paper, we will try to address some properties of steerable pyramid transform that are relevant for use in image watermarking; these properties include: (1) invariance properties; (2) multiresolution aspect; (3) capture of multi-scale and multiresolution structures in the image. All the above mentioned properties make this steerable pyramid transform appropriate for the design of a robust watermarking scheme.

This paper proposes an image watermarking scheme based on steerable pyramid transform to embed invisible and robust watermark. We can summarize the basic principles of our method as follow: a host image is first transformed by the steerable pyramid transform. The different features are then extracted by thresolding the different subbands. The watermark sequence is inserted into disjoint blocks centered on the extracted feature points. The original host image is needed in watermark detection mainly for extracting the featured coefficients necessary for robust detection and determining the value of one bit of the watermark spread into a block. It has been confirmed by experiments and comparisons with many existing non-blind techniques that the watermark information embedded by the proposed technique is robust to JPEG compression, additive noise, and median filtering.

Keywords : content based image watermarking scheme, steerable pyramid transform, edges, blocks, non-blind watermarking method.

1. INTRODUCTION

The rapid growth of networked multimedia systems and the ease of media manipulations gave rise to problems such as unauthorized copying and distribution. Digital watermarking recently emerged as an effective solution to these problems as it can preserve the copyright and data security.

The work presented in this article is concerned with the design of a robust watermarking algorithm with the steerable pyramid transform. We choose the steerable pyramid transform as watermarking domain because it has a lot of useful properties which can be investigated in image watermarking. Actually, steerable pyramid, similar to the other pyramid algorithms, is based on recursive application of filtering and subsampling operations but the steerable pyramid transform are addressed; these properties include: (1) invariance properties; (2) multiresolution aspect; (3) capture of multi-scale and multiresolution structures in the image. For instance, invariance properties of the steerable pyramid transform may be exploited to counterattack geometrical attacks. Robustness is also obtained by the multiresolution aspect resulting from the application of the steerable pyramid transform. This makes possible to hide watermarks with more energy in an image and also to insert many information's redundancies from one band to another. Moreover, the coefficients of the steerable pyramid transform can capture multi-scale and multi-orientation structures in the image. As such, we can calculate local orientations and then extract significant areas. Indeed, choosing to add our watermark into these significant areas –around edges and in textured areas- is very interesting: the human visual system is less sensitive to

distortions in these areas than in smooth ones. These features are extracted from each subband of the pyramid decomposition. All the above mentioned properties make this steerable pyramid transform appropriate for the design of a robust watermarking scheme.

We can summarize the basic idea of the method as follow: a host image is first transformed by the steerable pyramid transform. The different features are then extracted by thresolding the different subbands. The watermark sequence is inserted into disjoint blocks centered on the extracted feature points. The insertion is done with a redundant manner into the set of subbands resulting from the application of the steerable pyramid transform on the image. Consequently, each subband is individually marked. In watermark detection, original host image is needed mainly for extracting the featured coefficients necessary for robust detection and determining the value of one bit of the watermark spread into a block. It has been confirmed by experiments and by comparison with many existing non-blind techniques that the watermark information embedded by the proposed technique is robust to JPEG compression, additive noise and median filtering.

The rest of the paper is organized as follows: in section 2, an overview of the watermarking problem as well as comprehensive survey of watermarking techniques are briefly reviewed. The characteristics of the steerable pyramid transform is described in section 3. Its advantages over the conventional pyramid transforms are also highlighted. Section 4 deals with the application of this pyramid transform to image watermarking. This application faces the image copyright protection problem from the viewpoint of selecting only interesting image regions for embedding. This section focus on the description of our proposed method and presents experimental results. Finally, a conclusion is presented in section 5.

2. RELATED WORK ON DIGITAL IMAGE WATERMARKING

Digital watermarking is based on the science of data hiding [1]. The basic idea behind it is to embed an information in the host multimedia medium to assert its ownership. The insertion step may be performed in spatial [2,3], frequency [4,5,6] or multiresolution [7,8,9] domains, so that, if the embedded information can be reliably recovered, this information can specify the affiliation between the data and its owner. The watermark can then be extracted from the watermarked media to identify the owner. This step of signature verification can be developed in two different ways: using the original image or not. Watermarking techniques are not only used for protection purposes. Other applications include indexing, data hiding..[10]. Each watermarking application has its own specific requirements. Therefore, there is no set of requirements to be met by all watermarking techniques. Nevertheless, some general directions can be given for most of them. A watermarking framework must mainly satisfy a trade-off between two conflicting requirements: perceptual transparency and robustness to attack. The challenge here is to introduce a digital watermark that does not alter the perceived quality of the electronic content while being extremely robust to attacks. A very robust watermark can be obtained by making large modifications to the host data but such modifications will be violating the constraint of perceptual transparency. This problem can be solved by taking into account a model of the human visual system (HVS) which allows us to maximise the energy of the watermark while keeping its visibility to a minimum. According to the characteristics of the human visual system, a larger visual tolerance is normally applied for large edges and high activity textured regions. Hence, for image watermarking it is better to insert information in high activity regions rather than in smooth areas of an image. To do so, only the most significant components are marked. These components can be edges, corners, textured areas. This evoked the second generation watermarking [11]. Such watermarking methods may present additional advantages in terms of detection and recovery from geometric attacks. In fact, putting a watermark in the most important components of an image improves the robustness of the watermark, since tampering with these important image components to remove the watermark will severely degrade the quality of the image. Moreover, human eyes are less sensitive to noise in textured regions than in smooth areas of an image. Compression systems also tend to preserve textured areas and edges while coarsely quantising smooth areas. Based on these reasons, numerous approaches argue that one should embed watermarks in perceptually significant portions [12,13].

3. THE STEERABLE PYRAMID TRANSFORM : A BRIEF REVIEW

Recursive multi-scale transforms are now a standard tool in signal and image processing. However, their main drawback is the lack of translation invariance especially in two-dimensional (2-D) signals. To overcome this problem,

the steerable pyramid transform has been proposed by Freeman and Simoncelli [14]. In this linear decomposition, an image is subdivided into a collection of subbands of various orientations. The scale tuning of the filters is constrained by a recursive system diagram. The bloc diagram of this decomposition at frequency domain is shown in Fig. 1 which presents three types of filters: low-pass (L₀), high-pass (H₀), and pass-bands (B₀... B_K). In fact, both low-pass and high-pass filters separate the image into low and high pass subbands respectively. Then, the low-pass subband is further decomposed into K + 1 oriented subbands and a low-pass subband. The latter is then subsampled by a factor 2 and a new decomposition is performed until reaching the scale fixed by the user. Indeed, each successive level of the pyramid is constructed from the previous level's lowpass band. For instance, an example of one-level image decomposition can be found in Fig. 2, where four band-pass filters are involved.

Based on the comparison done on the steerable pyramid and two other well-known multi-scale representations (Laplacien pyramid and Dyadic QMF/wavelet) [14], we notice that the steerable pyramid retains some of the advantages of orthonormal wavelet transforms since its basis functions are localized in both space and spatial-frequency. Yet, this recursive multi-scale and multi-direction decomposition improves on some of their disadvantages. It is essentially aliasing-free and can be designed to produce any number of orientation bands as it is based on a class of arbitrary orientation filters generated by linear combination of a set of basis filters. One obvious disadvantage is in computational efficiency: the resulting transform is substantially overcomplete by a factor of 4(k + 1)/3. Indeed, the number of pixels in the pyramid is much greater than the number of pixels in the input image. The set of filters used in this linear decomposition are highly constrained. First of all, to ensure elimination of the aliasing terms, the filter L₁ should be band-limited, i.e.

$$L_1(w) = 0 \qquad for \qquad |w| \succ \pi/2 \tag{1}$$

Furthermore, to avoid amplitude distortion, the transfer function of the system should be equal to unity.

$$\left|H_{0}(w)\right|^{2} + \left|L_{0}(w)\right|^{2} \left|\left|L_{1}(w)\right|^{2} + \sum_{i=0}^{k-1} \left|B_{i}(w)\right|^{2}\right| = 1$$
(2)

Moreover, in order to cascade the system recursively, another constraint must be verified:

$$\left|L_{1}(w/2)\right|^{2} = \left|L_{1}(w/2)\right|^{2} \left|L_{1}(w)\right|^{2} + \sum_{i=0}^{k-1} \left|B_{i}(w)\right|^{2}\right]$$
(3)

The angular constraint on the band-pass filters B_k is determined by the condition of steerability and can be expressed as

$$B_{k}(w) = B(w) \left[-j\cos(\theta - \theta_{k}) \right]^{n}$$
(4)
Where for $\theta = \arg(w), \ \theta_{k} = k\pi/(n+1)$, $B(w) = \sqrt{\sum_{i=0}^{k-1} B_{k}(w)}$

Satisfying these conditions, Castleman proposed a simple method for designing the finite impulse response filter kernels required to implement the transform[15]. This method defines a one-dimensional low-pass (LP) and high-pass (HP) transfer function using the raised cosine.

$$LP(a,b,f) = \begin{cases} 1 & f \leq a \\ \sqrt{\frac{1}{2} \left[1 + \cos\left(\frac{f-a}{b-a} \cdot \pi\right) \right]} & a \prec f \prec b \\ 0 & f \geq b \end{cases}$$
(5)

$$HP(a,b,f) = \begin{cases} 0 & f \leq a \\ \sqrt{\frac{1}{2} \left[1 + \cos\left(\frac{f-a}{b-a} \cdot \pi\right) \right]} & a \prec f \prec b \\ 1 & f \geq b \end{cases}$$
(6)

Where a and b are the parameters that establish band limits. Concerning the steerable bandpass filters, the k directional bandpass filters used in the iterated stages are given by:

$$B_{m}(\theta) = HP(f_{1}, f_{N}/2, f) \frac{\left(-i\cos\left(\theta - \frac{n\pi}{K}\right)\right)}{\sqrt{\sum_{n=0}^{K-1}\cos(\frac{n\pi}{K})^{2(K-1)}}}$$
(7)

The steerable pyramid has been found useful in a variety of image processing applications such as orientation analysis, noise removal and enhancement [17], transient detection, texture synthesis, contour detection [16].



(b) Spectral decomposition performed by a steerable pyramid with four directions (B₁... B₄).

Fig. 1. Graphical illustration of the steerable pyramid transform



Fig. 2. Example of steerable pyramid-based image decomposition

4. APPLICATION TO IMAGE WATERMARKING

The general approach used in many existing papers is to divide the image into blocks. Each block is mapped into either the spatial domain or the transform domain or both of them; in other words the information is embedded in the spatial and frequency domain [18]. Spatial domain based methods embed messages directly in the intensity of pixels. As a block based spatial watermarking method, we can cite the work of Darmstaedter et al. [19]. This method tries to adapt the embedding rule with regards to the content of blocks. Consequently, many perceptual tests have been achieved in order to determine the optimal solution. For frequency-domain based ones, images are first transformed to frequency domain, and then messages are embedded in the transform coefficients. For instance, Jnag et al. [20] proposed a digital watermarking algorithm in which the watermark information is embedded only into the selected complex blocks. These 8x8 DCT blocks are selected by calculating their AC coefficients. Our approach investigates a block based image watermarking technique. But, rather than dividing into blocks, we propose to establish 3x3 non-overlapping blocs of neighboring coefficients for each edge pixel extracted. The main idea of using blocks here is to spread the information among many pixels of an image since the steerable pyramid transform alter enormously the value of pixels once these values are modified, reconstructed then decomposed again. The block size is chosen very small in order to make changes very close to the edges and to keep more important the strength of the watermark without being perceptible. By doing so, we need to perform an edge extraction procedure from steerable pyramid transform coefficients.

4.1. WATERMARK EMBEDDING SCHEME

The general embedding method can be decomposed into different steps. First of all, the host image is transformed into steerable pyramid coefficients by using specific values of scales and directions determined by the user. Edges and high textured area must then be extracted. Indeed, with the steerable pyramid transform, edges and textures are usually well confined to larger magnitude coefficients in the different subbands. Consequently, high values are selected by using a threshold. The threshold value is set equal to the maximum of the considered subband divided by a factor 3. The next step is to find out 3x3 disjoint blocks included in thresholded edges. Then a watermark sequence is inserted in all these blocks. The number of bits that may be encoded equals the number of generated blocks and this in turn depends on the size of the image and on its homogeneity. Hence, more homogenous the image is, less the extracted feature are and vice versa. The redundancy in the code can be adequate for some applications such as for error detection and correction. Using this may introduce a high amount of redundancy for the tag information especially when the number of 3X3 blocks extracted from a subband is higher than the watermark's length. This redundancy can be achieved not only through each level of the pyramid but also in each subband. For embedding the information, each bit is spread into one block. All the values of the blocks are modified as follows :

$$X_{w,k} = X_k + \alpha W_k \tag{8}$$

where $X_{w,k}$ is the watermarked coefficient and X_k is the original coefficient with the coordinate k in the spatial position. W_k is the corresponding watermark symbol, which takes two values -1 and 1. The weighting factor α is a positive number which determines the strength of the watermark. It is used to adjust the amount of added watermark energy. The value of alpha is adjustable by the user to achieve a balance between robustness and fidelity of the resulting watermarked image. This factor also depends on the treated image. Doing this, the mean of each block may then be incremented to encode a '1' or decremented to encode a '0'. Finally, the inverse pyramid transform is then applied combining the different watermarked subbands to form the watermarked image.

4.2. WATERMARK EXTRACTION SCHEME

The detection process needs the original image for extracting the signature as well as the presence of the watermarked image. Original host image is needed in watermark detection mainly for extracting the featured coefficients necessary for robust detection and determining the value of one bit of the watermark spread into a block. The general extraction method can be decomposed into different steps. First of all, the host image and the watermarked one are transformed into the steerable pyramid coefficients with the same parameters as in the embedding stage. In order to localize the marked block, the same rule described latter is also available here. Then, the extraction of all signature occurrences is based on a comparison between the blocks of the two available images. Finally, we succeed to extract correctly the signature embedded by averaging all the occurrences.

4.3. EXPERIMENTS

To prove the effectiveness of the proposed copyright protection scheme, we conduct series of experiments performed with real images. The first test images include "Lena", "Baboons" and "Cameraman" image of size 256x256 and a 119 bits signature, consisting in a logo image. By using the proposed method, the watermark is almost invisible to the human eyes, as shown in Fig.4. In fact, the choice of the strength of the inserted watermark must assume no distortions inserted. Then, a high parameter (*alpha*) can affect the visual quality of the image (Fig.5).



This value previously defined in section 4.1 represents the weighting factor. For example, a value of 20 is used in the tests in order to guarantee no visual degradation of the watermarked image. But, generally, this parameter depends on the original image. Moreover, as we previously noticed, the embedding scheme is realized with many redundancies of the signature. In order to estimate this number, we established many measures for different values of scales and directions numbers and for different images. It appears (Tab.1) that the signature occurrences vary from one image to another and from one parameter (scale, direction) to another. In particular, highly-textured images can be watermarked with many

redundancies. The redundancy has many advantages especially in improving the quality of the extracted signature (Fig.6) or increasing robustness toward cropping attacks (Fig.7).

		Watermark redundancies								
Number of scales		1			2			3		
Number of directions		1	2	3	1	2	3	1	2	3
Image	Mandrill	5	11	13	7	12	13	7	12	13
	Cameraman	2	3	7	3	3	9	3	3	9
	Lena	3	3	6	4	4	7	4	4	7

Tab.1. 119 bits signature redundancies



A second series of experiments have been carried out to verify the robustness of the scheme. Different kinds of image processing were used to check whether the embedded watermark was transparent and robust. The list of the modifications treated here represents in no way an exhaustive overview. It just gives a hint on the current operations achieved on images. By doing so, we have compared the proposed method with other non-blind existing frequency-domain techniques. These techniques have been studied and implemented by Meerwald [21]:

- Cox et al.[6] proposed a secure spread spectrum watermarking scheme and the watermark is embedded using the largest amplitude DCT coefficients.

- Wang et al.[22] developed a new frequency-domain wavelet-based watermarking technique embedding information into significant coefficients across subbands.

- The work of Kim et al.[23] deals with a wavelet based multiresolution watermarking scheme using the human visual system (HVS). Pseudorandom codes are added to the large coefficients which are selected in each DWT band of an image.

- Corvi et al.[24] described an embedding strategy based on the work of Cox et al. that operates on the coefficients of a low-resolution representation of the host image. As such all coefficients of the LL subband are watermarked.

We choose to insert a random binary sequence of 119 bits length and to achieve simulation on a 256x256 Lena image. Tab.2 enumerates the different values of PSNR which guarantee no visual degradation of the watermarked image obtained after inserting watermark using the different methods.

	Cox's method	Kim's method	Corvi's method	Wang's method	Proposed method
PSNR	40	47	41	35	38

Tab.2. PSNR obtained by the different tested methods

All the results depend on the correlation computation. A correlation detector is used to calculate the similarity between the original watermark sequence and the extracted one. To calculate it, we use the vector projection as defined by:

$$corr (X, X') = \frac{X \cdot X'}{\sqrt{X' \cdot X'}}$$
⁽⁹⁾

where X is the original watermark and X' is the extracted watermark.

Before starting the comparisons, optimal values of scales and directions that define the parameters of the steerable pyramid decomposition must be fixed. Consequently, we test the robustness of the method under median filtering and JPEG compression attacks with various number of scales and directions. The robustness is evaluated using the above mentioned correlation detector (see equation 9). From these tests (Fig.7, Fig.8), we choose the couple (scales=3, directions=2) as it gives approximately the most interesting results. Following the choice of the precedent couple, comparison of the robustness of various methods under different attacks have been carried out. The considered attacks are : (1) 35dB Gaussian noise(Fig.9); (2) Histogram equalisation(Fig.9); (3) cropping(Fig.9); (4) median filtering(Fig.11); (5) salt and pepper noise(Fig.12); (6) JPEG compression(Fig.13). For each attack, the figures compare the robustness when using the methods: Cox, Kim, Corvi, Wang and the proposed method with a two directions and three scales decomposition.

From these tests, it appears that:

- **under no attack**(Fig.9), the watermark detection is almost the same for all the considered methods and the value of correlation is close to unity. The watermark detection is good.
- under 35 dB Gaussian noise attack(Fig.9), the correlation values for the different methods are almost the same. This is a prove of a good robustness.
- **under salt and pepper noise attack**(Fig.12), the plotted curves show a variation of the correlation value as a function of different salt and pepper rates. We notice that results associated to our method and Wang's method are the best ones.
- under histogram equalisation attack(Fig.9), the proposed method provides higher correlation values compared to the others.
- **under cropping attack**(Fig.9), we do not consider the entire original image for watermark detection but we a part of it (60%) (Fig.10). Figure 9 proves the outperformance of this method applied to a fraction of the image.
- **under median filtering attack**(Fig.11), the plotted curves show a variation of the correlation value as a function of of the median filter window size. We notice that the results associated to the proposed method are good enough to justify the robustness under this attack.
- **under JPEG compression attack**(Fig.13) we considered tests with different compression rate. By varying the peak signal to noise ratio (PSNR) (Fig.14), we notice that for PSNR values exceeding 34 dB, the proposed method achieves the higher correlation values, while for lower PSNR, results are similar to those obtained with Cox's and Corvi's methods.

By interpreting the results of the different figures described above, we can conclude that the robustness of the proposed method outperform the other compared methods : Cox, Kim, Wang and Corvi.







5. SUMMARY AND FUTURE WORK

The application of the steerable pyramid transform in image watermarking has been presented. The proposed method embeds invisible and robust watermark into significant areas –around edges and in textured areas- of an image. The human visual system is less sensitive to distortions in these areas than in smooth ones. The watermark is added to each subband of a steerable pyramid decomposition. Our results show good robustness to JPEG compression, median filtering, additive noise, cropping and histogram equalisation. Although there is still a lot of work to do, the results are promising. Additional efforts on both the theoretical and the practical side need to be done on at least the following points:

- automate the choice of some parameters especially watermark strength, number of scales and directions which will guarantee high degree of robustness and best performances.

- Coming upon a blind image watermarking scheme i.e. extracting the embedded watermark without the need of the original image.

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